6-13-2008

Minnesota Department of Education

June 12, 2008

Legislative Reference Library
645 State Office Building
100 Constitution Avenue
St. Paul, Minnesota 55155

Re: In The Matter of the Proposed Permanent Rules of the State Department of Education Governing Mathematics Academic Standards, *Minnesota Rules*, 3501.0700-3501.0745, and Repeal of *Minnesota Rules*, 3501.0560, 3501.0565, 3501.0570, 3501.0575, 3501.0580, 3501.0585, 3501.0590, 3501.0595, 3501.0600, 3501.0605, 3501.0610; Governor's Tracking #AR 387

Dear Librarian:

The Minnesota Department of Education intends to adopt rules relating to mathematics academic standards. We plan to publish a Dual Notice in the June 16, 2008 State Register.

The Department has prepared a Statement of Need and Reasonableness. As required by Minnesota Statutes, sections 14.131 and 14.23, the Department is sending the Library a copy of the Statement of Need and Reasonableness at the same time we are mailing our Notice of Intent to Adopt Rules.

If you have questions, please contact me at (651) 582-8669.

Cordially,

Kathryn Olson.

Kathryn Olson
Rulemaking Coordinator

Enclosure: Statement of Need and Reasonableness

Minnesota Department of Education

Proposed Permanent Rules Relating to Academic Standards for Mathematics, *Minnesota Rules*, 3501.0700-3501.0745, and Repeal of *Minnesota Rules*, 3501.0560, 3501.0565, 3501.0570, 3501.0575, 3501.0580, 3501.0585, 3501.0590, 3501.0595, 3501.0600, 3501.0605, 3501.0610.

Introduction

The state of Minnesota has implemented statewide academic standards for Minnesota public schools since 1997, starting with the Profile of Learning standards. The establishment of statewide academic standards in core content areas was first required by the Elementary and Secretary Education Act (ESEA) re-authorization that occurred in 1994. In 2003, the Minnesota Legislature repealed and replaced the Profile of Learning with grade-specific standards in reading and mathematics in order to comply with the re-authorization of ESEA, now widely known as the *No Child Left Behind Act of 2001*. Pub. L. 107-110. In 2004, the Minnesota Legislature adopted new state standards in science and social studies. *See* 2004 Minnesota Laws, chapter 294, article 2, section 2.

Legislation passed in 2006 requires that Minnesota's academic standards and high school graduation requirements in mathematics be revised to reflect an increased level of rigor that prepares students with the knowledge and skills needed for success in college and the skilled workplace. The legislation also establishes a timetable and requirements for revising state academic standards in each subject and directs the Minnesota Department of Education (Department) to revise these state academic standards. *See* Minn. Stat. § 120B.023, subd. 2.

The legislation is a response to economic, demographic and student performance trends that indicate an urgent need for higher mathematics achievement for most, if not all, of Minnesota's students. In the future, all Minnesota students will need some level of postsecondary education. Mastery of rigorous high school mathematics content is a prerequisite for postsecondary success and access to the high-wage and high-demand jobs of the future.

Minnesota's new high school graduation requirements reflect the need for more rigorous mathematics standards. Students in the class of 2007-08 and thereafter, must complete three high school credits in mathematics encompassing the algebra, geometry, statistics and probability content specified in the 2003 mathematics standards. In addition to these current requirements, legislation passed in 2006 requires all students to take Algebra I or its equivalent by the end of Grade 8. Furthermore, all high school students must successfully complete Algebra II or its equivalent as a condition for graduation. These two new requirements are effective for the graduating class of 2015 and thereafter. In addition to course credits and content requirements, students must pass a state mathematics test. Students graduating in 2009-10 must reach proficiency on the state Grade 11 MCA-II mathematics assessment or pass the Grade 11 Graduation-Required Assessment for Diploma (GRAD) assessment in mathematics. Students will have additional opportunities to retake the GRAD test in Grades 11 and 12.

In response to the new graduation requirements, the legislature called for the revision cycle for mathematics standards to begin in the 2006-07 school year. *See* Minn. Stat. § 120B.023, subd. 2. The new standards take effect in the 2010-11 school year and must reflect the following:

- Algebra I by the end of Grade 8 and Algebra II in high school for graduation,
- Technology and information literacy standards, and
- College and work-readiness skills and knowledge.

The Department facilitated the Mathematics Standards Revision Committee (Committee), a group consisting of K-12 mathematics teachers, postsecondary mathematics instructors, business and community representatives and parents. Applications for the committee were submitted online and the commissioner selected 26 applicants. Two co-chairs were named, both of whom had leadership roles in the development of the 2003 mathematics standards. In addition to knowledge of mathematics content and pedagogy spanning the K-12 Grade levels, members brought to the Committee expertise that included teaching students with special needs, English Language Learners, and low-income, and urban and rural students. Parents, business and higher education faculty were also represented on the committee. See attachment for a list of members of the Mathematics Standards Revision Committee.

The Standards Revision Committee met from October 11, 2006, through March 30, 2007. Several members of the committee served on the Technical Writing Team. The Committee met nine times to review feedback and provide direction to the Technical Writing Team. The Technical Writing Team met in between meetings of the full committee and revised the draft according to direction provided by the Committee.

The Department invited the public to submit suggestions for revising the standards through an online process that was completed prior to the first meeting of the Committee. The feedback was collected, sorted into categories of like suggestions, and submitted to the Committee. After careful consideration of the online feedback, standards from other states, national frameworks documents and national reports, the Committee prepared a draft of the revised mathematics standards. The Department solicited feedback on the draft revisions from a number of sources in the following ways:

- The public was invited to submit online feedback regarding the first draft of the revised standards.
- The public was invited to ask questions and submit comments at regional meetings hosted by Commissioner Alice Seagren, Deputy Commissioner Chas Anderson, former Assistant Commissioner Mary Ann Nelson and other Department staff. The meetings were held in November and December in Rochester, Mankato, Marshall, Fergus Falls, St. Cloud, Duluth, Bemidji and Roseville.
- The Commissioner hosted small meetings with representatives from the Minnesota Council of Teachers of Mathematics, SciMath Minnesota and other groups of interested stakeholders.
- The Department convened a team of special education professionals to review the draft standards for items that might be biased against students with special needs.

Finally, the Department solicited detailed feedback from several reviewers widely considered to be experts in K-12 standards and mathematics education. The expert reviewers included the following:

- Dr. Sybilla Beckmann-Kazez, Department of Mathematics, University of Georgia;
- Dr. Lynn Steen, Department of Mathematics, St. Olaf College;
- Dr. David Klein, Department of Mathematics, California State University – Northridge;
- Dr. Steven Leinwand, American Institutes for Research; and
- Achieve, Inc., as part of Minnesota's participation in the American Diploma Project network.

The Committee considered the feedback from all of the sources listed above in making its final changes to the draft. A college/work readiness study from Achieve and the Department's own work with the American Diploma Project network and Achieve were also instrumental in producing a final draft. The Commissioner shared the final draft of the mathematics standards with the legislature at a joint Senate/House Education Policy Committee hearing in May 2007.

The need for more rigorous mathematics standards for our students is clear, particularly in light of job growth projections and the kinds of knowledge and skills that students will need to succeed at those jobs. Education policy analysts Charles Coble and Michael Allen framed the issue in the context of the global economy when they wrote "America's competitive edge in the global economy, [and] the strength and versatility of its labor force…are increasingly dependent on an education system capable of producing a steady supply of young people well prepared in science and math."[1]

Nationally, the science, technology, engineering and mathematics (STEM) occupations had earnings that were about 70% more than the national average in 2005. Today, every major group of STEM occupations enjoys overall median earnings that are above the national average. These findings are important because higher than average earnings are often an indicator of strong demand for workers.[2]

In fact, the national demand for workers with strong STEM skills has been evident for some time. Over the last two decades, the U.S. science, engineering and technology workforce has grown at more than *four times* the rate of total employment. Employment projections for 2000-2010 reveal that over 80% of the fastest-growing occupations and two-thirds of the occupations with the largest job growth are dependent upon a knowledge base in science and mathematics.[3]

Likewise, job growth in Minnesota is most apparent in the STEM areas. The average growth rate for all occupations in the state from 2006-2016 is projected to be 9.8%. Economists predict that over the next ten years, many jobs in STEM-related fields or those requiring some degree of STEM competency will outpace the average growth rate. For example, occupations in network

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[1] Charles Coble and Michael Allen, *Keeping America Competitive: Five Strategies To Improve Mathematics and Science Education*, Education Commission of the States, 2 (July 2005).
[2] Bureau of Labor Statistics, Occupational Outlook Quarterly, (Spring 2007).
[3] Charles Coble and Michael Allen, *Keeping America Competitive: Five Strategies To Improve Mathematics and Science Education*, Education Commission of the States, 2 (July 2005).

systems and data communications analysis will grow an astounding 51.4%; computer software engineering and applications will increase 40.6%; and registered nursing and dental hygienist jobs will grow 29.4% and 27.6% respectively.[4]

There are indications that Minnesota may not be able to supply the workers needed to fill these high-growth and high-paying jobs. Only 10.8% of 8th graders and 20.9% of 9th graders in Minnesota have expressed an interest in pursuing a future in science, technology, engineering and mathematics fields.[5] Further, the number of students awarded degrees in the STEM fields from Minnesota postsecondary institutions is, by four different measures, *below the national average*.[6]

The "supply" problem is exacerbated by the need for more students to complete some level of postsecondary education to meet the increased skill requirements of high-growth jobs, and to offset the large number of "baby boomer" workers expected to retire soon. One organization estimates that more than two-thirds of the new jobs created by 2012 will require some education beyond high school.[7] Examples of postsecondary education required by these new jobs span a range of options including short-term technical training, a two-year associate's degree or industry credential, or a bachelor's degree. Beginning in 2012, Minnesota will need an additional 10,500 college graduates per year to meet future workforce demands. At the same time, an estimated 9,000 to 25,000 college graduates currently in the workforce will be retiring each year.[8] As a result, Minnesota's K-12 education system needs to prepare many more students to attend postsecondary institutions.

The performance of Minnesota students on national and international assessments reveals the extent to which students currently are prepared for postsecondary mathematics. Evidence from the National Assessment for Educational Progress (NAEP) and the ACT college admissions test indicate that the current Minnesota mathematics standards enable many students to perform as well or better than students in most other states. However, while the state shows an overall strong performance, two important trends must be taken into account:

- Some student groups are consistently under-performing on several measures. Students of color and low-income students take fewer high school mathematics courses, perform less well on assessments, and graduate from high school and college at lower rates.[9]
- Minnesota's overall strong achievement pales in comparison to the achievement of the top performing countries. The state participated as a mini-nation in the 1995 Third

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[4] Minnesota Department of Employment and Economic Development 2006-2016 job growth rate projections. Accessed 4/24/08 at http://www.deed.state.mn.us/lmi/tools/projections/Default.aspx.

[5] Minnesota Department of Education. American College Testing (ACT) Education Planning & Assessment System (EPAS) 2006 survey results of Minnesota 8th and 10th graders. http://education.state.mn.us.

[6] Minnesota Office of Higher Education, *Minnesota Measures: 2007 Report on Higher Education Performance* 24-25 (2007).

[7] Achieve, Inc. *Case for Action Toolkit.* Accessed 10/17/07 at http://www.achieve.org/node/329.

[8] Minnesota Private College Research Foundation, *Projections of High School Graduates: Implications for Baccalaureate Degree Production and Workforce Growth.* iii (April, 2004). See entire report at http://www.mnprivatecolleges.org/userFiles/File/Research/hs_grad_projections.pdf.

[9] Minnesota State Demographic Center. *Minnesota Educational Trends 2000-2005.* Accessed 11/06/2006 at http://www.demography.state.mn.us.

International Mathematics and Science Study (TIMSS). Minnesota students achieved better scores than the U.S. average in math, but well below the highest performing countries.[10]

Students who perform well on these assessments typically have taken rigorous mathematics courses in high school. Those who have not taken such courses are much more likely to end up in remedial courses in college and are more likely to drop out altogether.[11] Unfortunately, as of 2005, Minnesota is lagging behind the national average in the percentage of 8th graders taking algebra. Only 32% of Minnesota 8th graders took algebra compared to 41% of 8th graders nationally. In top performing states, 56% of 8th graders took algebra.[12] Additionally, student mathematics knowledge is not matching up with college expectations. Mathematics is the most common remedial course taken in Minnesota's public colleges. Thirty-six percent of students entering public higher education institutions in the class of 2005 took at least one remedial course in mathematics within two years of high school graduation.[13]

Students who complete rigorous mathematics courses in high school have a much greater chance of graduating from college. Of all pre-college curricula, the highest level of mathematics a student completes in secondary school has the strongest continuing influence on bachelor's degree completion.[14] Finishing a course beyond the level of Algebra II, for example, trigonometry or pre-calculus, more than doubles the odds that a student who enters postsecondary education will complete a bachelor's degree.[15] Not surprisingly, rigorous mathematics standards are needed for all students so that they will be academically prepared for the postsecondary education required by jobs in high-wage and high-demand fields.
While much of this discussion has focused on jobs and economic competitiveness, it also should be noted that the safety of our society and the quality of life—not just the prosperity of the state and nation—are of concern. For centuries, mathematics education outcomes have been intimately linked to the ability of people to deal with sophisticated quantitative ideas. Mathematical skills are the foundation for advances in medicine and health, in technology and commerce, in navigation and exploration, in defense and finance, and in the ability to understand past failures and to forecast future developments.[16]

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[10] Developing World Class Students through World Class Mathematics Standards: Do Minnesota's Standards, Students, and Teaching Measure Up? SciMathMN Briefing for State Policymakers, January 2007. http://www.scimathmn.org/scimathweb2/scimweb2_pdfs/World_Class_Standards.doc.pdf (Accessed 10/16/2007).
[11] Achieve, Inc. *Case for Action Toolkit.* (Accessed 10-17-2007). http://www.achieve.org/node/329.
[12] *Are students taking "gateway" courses?* Achieve Data Profile: Minnesota. American Diploma Project Network, Achieve, Inc. (May 2007). http://www.achieve.org/files/Minnesota05-08-07.ppt#661,12.
[13] *Getting prepared: A 2008 report on recent high school graduates who took developmental/remedial courses.* Minnesota State Colleges & Universities and the University of Minnesota (March 2008). http://www.mnscu.edu/media/publications/pdf/gettingprepared05.pdf (Accessed 10/16/2007).
[14] Carnevale, Anthony P. and Desrochers, Donna M. *Standards for What? The Economic Roots of K-16 Reform.* Educational Testing Service (2003). Available at http://www.transitionmathproject.org/assets/docs/resources/standards_for_what.pdf (Accessed ???).
[15] Adelman, Clifford, 1999. *Answers in the Toolbox: Academic Intensity, Attendance Patterns, and Bachelor's Degree Attainment.* U. S. Department of Education. http://www.ed.gov/pubs/Toolbox/toolbox.html. Accessed 11/6/2006
[16] National Mathematics Advisory Panel, *Foundations for Success: The Final Report of the National Mathematics Advisory Panel,* U.S. Department of Education (March 2008).

The recently released report of the National Mathematics Panel captures the magnitude of the problem:

> During most of the 20th century, the United States possessed peerless mathematical prowess—not just as measured by the depth and number of the mathematical specialists who practiced here but also by the scale and quality of its engineering, science, and financial leadership, and even by the extent of mathematical education in its broad population. But without substantial and sustained changes to its educational system, the United States will relinquish its leadership in the 21st century.

By revising the K-12 mathematics standards and graduation requirements, the state is making substantial and sustained improvements in the mathematics education of all Minnesota students.

Statutory Authority

In 2006, the legislature gave the Department general rulemaking authority to revise and align the state's academic standards and high school graduation requirements in mathematics, arts, science, language arts and social studies beginning in the 2006-07 school year and continuing through the 2019-2020 school year. Minn. Stat. § 120B.023, subd. 2. The legislature directed the Department to revise the mathematics standards to include Algebra I or its equivalent credit by the end of Grade 8 and satisfactory completion of Algebra II or its equivalent for graduation. Specifically, the legislation provides that:

> (b) The commissioner in the 2006-07 school year must revise and align the state's academic standards and high school graduation requirements in mathematics to require that students satisfactorily complete the revised mathematics standards, beginning in the 2010-11 school year. Under the revised standards:
> (1) students must satisfactorily complete an algebra I credit by the end of eighth grade; and,
> (2) students scheduled to graduate in the 2014-15 school year or later must satisfactorily complete an algebra II credit or its equivalent.

Minn. Stat. § 120B.023, subd. 2(b).

Alternative Format

Upon request, the Statement of Need and Reasonableness can be made available in an alternative format. To make a request, contact Gloriann McDonald at the Minnesota Department of Education, 1500 Highway 36 West, Roseville, MN 55113; phone 651-582-8734. TTY users may call the Minnesota Department of Education at 651-582-8201.

Regulatory Analysis

Minnesota Statutes § 14.131, sets out seven factors for a regulatory analysis that must be included in the SONAR. Paragraphs (1) through (7) quote these factors followed by the agency's response.

(1) A description of the classes of persons who probably will be affected by the proposed rule, including classes that will bear the costs of the proposed rule and classes that will benefit from the proposed rule.

The following classes of persons affected by proposed rule: Minnesota students graduating from high school in the year 2015 and beyond; Minnesota school districts; and Minnesota charter schools.

(2) The probable costs to the agency of the implementation and enforcement of the proposed rule and any anticipated effect on state revenues.

The proposed rules do not create any additional costs to the Department until FY 2011. The Department is already staffed to provide training and support regarding the proposed rules and staff assignments until FY 2011. The legislature provided a one-time appropriation of $3 million to deliver regional services for mathematics and science teacher education. The Department will seek to have this funding made permanent in FY 2011 and beyond and resources will be reallocated accordingly.

(3) A determination of whether there are less costly methods or less intrusive methods for achieving the purpose of the proposed rule.

Because state standards in mathematics are both a federal and state requirement, there is no alternative method for achieving the purpose of the proposed rule. The proposed rules simply revise existing rules.

(4) A description of any alternative methods for achieving the purpose of the proposed rule that were seriously considered by the agency and the reasons why they were rejected in favor of the proposed rule.

Because state standards in mathematics are both a federal and state requirement, there is no alternative method for achieving the purpose of the proposed rule. The Department did not consider drafting the proposed rules without utilizing the knowledge and practical expertise of many different viewpoints and stakeholders.

(5) The probable costs of complying with the proposed rule, including portion of the total costs that will be borne by identifiable categories of affected parties, such as separate classes of governmental units, businesses, or individuals.

School districts may face initial increased costs to implement the new rules. However, districts currently must implement mathematics standards in Grades K-12. In addition,

school districts typically undertake a six- or seven-year curriculum adoption cycle, so many of these costs would be borne regardless of the new adoption of mathematics standards.

(6) The probable costs or consequences of not adopting the proposed rule, including those costs or consequences borne by identifiable categories of affected parties, such as separate classes of government units, businesses, or individuals.

If the state does not adopt mathematics standards, it risks the loss of federal funding. Section 1111(g)(1) of the *No Child Left Behind Act*, Pub. L. 107-110, states that for failure to meet deadlines enacted in 1994, in general,

> If a State fails to meet the deadlines established by the Improving America's Schools Act of 1994 (or under any waiver granted by the Secretary or under any compliance agreement with the Secretary) for demonstrating that the State has in place challenging academic content standards and student achievement standards, and a system for measuring and monitoring adequate yearly progress, the Secretary shall withhold 25 percent of the funds that would otherwise be available to the State for State administration and activities under this part in each year until the Secretary determines that the State meets those requirements.

Furthermore, section 1111(g)(2), states that for failure to meet the requirements enacted in 2001, "the Secretary may withhold funds for State administration under this part until the Secretary determines that the State has fulfilled those requirements."

(7) An assessment of any differences between the proposed rule and existing federal regulations and a specific analysis of the need for reasonableness of each difference.

The federal government requires states to have rigorous grade-specific academic standards in reading, math, science and social studies. It is up to each to state to develop the standards and the federal government reviews the standards and state assessment system. The current standards, adopted in 2003, have been approved by the federal government. Since the mathematics standards provide more rigor, it is expected that the federal government will also approve the proposed mathematics standards.

Cost to Small Businesses and Small Cities

As required by Minn. Stat. § 14.127, the Department has considered whether the cost of complying with the proposed rules in the first year after the rules take effect will exceed $25,000 for any small business or small city. The Department has determined that the cost of complying with the proposed rules in the first year after the rules take affect will not exceed $25,000 for any small business or small city.

This determination was made because the proposed rules do not affect small businesses and small cities.

Consult with Finance on Local Government Impact

As required by Minn. Stat. § 14.131, the Department has consulted with the Commissioner of Finance. On April 28, 2008, prior to the Department publishing the Notice of Intent to Adopt, the documents that were sent to the Governor's office for review and approval were also sent to the Commissioner of Finance. The documents included the Governor's Office Proposed Rule and SONAR Form; final proposed rules; and Statement of Need and Reasonableness. In a May 8, 2008 memorandum, the Department of Finance stated that the proposed rule will have little fiscal impact on local units of government.

Performance-Based Rules

Throughout the development of the proposed rules and this SONAR, the Department made every attempt to develop rules that will be understandable to practitioners and families ensuring efficient and effective delivery of services while achieving the best possible results for students.

Additional Notice

Minnesota Statutes, sections 14.131 and 14.23 requires that the SONAR contain a description of the Department's efforts to provide additional notice to persons who may be affected by the proposed amendments to the rules.

In addition to mailing the proposed rules and the dual notice to all persons who have registered to be on the Department's rulemaking mailing list under Minn. Stat. § 14.14, subd. 1a, the Additional Notice Plan calls for notifying the following groups:

- Individuals and organizations on the agency's registered rulemaking list;
- Education organizations list maintained by the agency;
- Attorneys lists maintained by agency;
- Minnesota superintendents, via the agency's weekly superintendent's informational email;
- Charter school directors via email lists maintained by the agency;
- Minnesota Association of Colleges of Teacher Education;
- SciMath Minnesota;
- Mathematics and Science Teacher Academy Advisory Task Force;
- Mathematics Standards Revision Committee;
- Minnesota Council of Teachers of Mathematics;
- Other interested parties; and
- Posting on the agency's website.

The Department also will send a press release to news outlets, including radio, television and newspapers throughout the state. The press release will describe the proposed rules; the Department's intent to adopt those rules unless it receives 25 or more requests for a hearing; and information about the hearing and cancellation of the hearing if it receives fewer than 25 requests for a hearing.

Finally, the Department will notify the Minnesota Legislature. This will include sending the proposed rules, SONAR, and Notice of Intent to Adopt to the chairs and ranking minority members of the legislative policy and budget committees with jurisdiction over the subject matter.

List of Witnesses

If these rules go to a public hearing, the Department anticipates having the following witnesses to testify in support of the need for and reasonableness of the rules:

1. **Karen Klinzing, Assistant Education Commissioner, Minnesota Department of Education.**
   Ms. Klinzing will testify on the need for the proposed mathematics standards from the state perspective. Ms. Klinzing is responsible for oversight of the academic standards at the Minnesota Department of Education. She previously served as a Minnesota legislator and classroom teacher in Bloomington Public Schools.

2. **Ellen Delaney, Associate Principal, Spring Lake Park High School.**
   Ms. Delaney was co-chair of the Mathematics Standards Revision Committee convened in 2006 and 2007. Ms. Delaney previously served as a mathematics teacher and curriculum director in North St. Paul-Oakdale-Maplewood Public Schools. Ms. Delaney will testify on the proposed mathematics standards relative to the development process, and the rationale and need for the new standards from the K-12 system perspective.

3. **Larry Gray, Ph.D., Head of Mathematics Department, University of Minnesota.**
   Dr. Gray was co-chair of the Mathematics Standards Revision Committee convened in 2006 and 2007. Dr. Gray will testify on the proposed mathematics standards relative to the development process, and the rationale and need for the new standards from the perspective of higher education.

4. **Peter Lindstrom, Vice President for Public Affairs at the Minnesota High Tech Association.**
   Mr. Lindstrom will testify on the need for the proposed mathematics standards from the perspective of Minnesota businesses.

5. **Kathryn Olson, Rulemaking Coordinator, Minnesota Department of Education.**
   Ms. Olson will facilitate the rule hearing process.

Rule-by-Rule Analysis

A significant change recently enacted by the legislature calls for the Minnesota Academic Standards in Mathematics to reflect a new requirement for students to complete Algebra I or its equivalent by the end of Grade 8. The Department began its rule revision process by devising Grade 8 standards that include and support the Algebra I requirement. The proposed revision of the Minnesota academic standards requires that all students are provided with learning experiences that will enable them to learn significant mathematics, and be prepared for and successfully complete the new Grade 8 Algebra I requirement. The Department has determined that the changes to the Grade 8 rules are necessary to support Algebra I in Grade 8, and that corresponding changes to the standards at earlier grades are warranted to support the change at Grade 8. The 2003 mathematics standards are not in compliance with these requirements and are hereby repealed.

REPEALER. Minnesota Rules, parts 3501.0560, 3501.0565, 3501.0570, 3501.0575, 3501.0580, 3501.0585, 3501.0590, 3501.0595, 3501.0600, 3501.0605 and 3501.0610 are repealed.

Minnesota state law requires academic standards and benchmarks for mathematics in Grades K-12.[17] Academic standards describe the expectations in mathematics that all students must satisfy to meet state requirements for credit and graduation. In order to measure whether these expectations are being met, the statewide Minnesota Comprehensive Assessments (MCA-II) are based on the standards, and all mathematics standards must be tested each year in Grades 3-8 and also in Grade 11. The benchmarks supplement the academic standards, and provide details about "the academic knowledge and skills that schools must offer and students must achieve to satisfactorily complete" the standards. Minn. Stat. § 120B.023, subd. 1. This SONAR will refer to the benchmarks from time to time in discussing the proposed standards changes, when the benchmarks play a role in those proposed changes.[18]

The proposed standards for Kindergarten through Grade 7 were revised to assure that background knowledge is provided to support this change to the Grade 8 Algebra I requirements. In addition, the Committee paid particular attention to the need for coherence in the standards from grade level to grade level. Dr. William Schmidt, University Distinguished Professor at Michigan State University, and a recognized leader in the area of educational policy related to mathematics, science and testing, characterizes curriculum in the United States as "a mile wide, and an inch deep" – a reference to the wide range of topics and lack of depth in mathematics curricula in the United States. According to Schmidt, this phenomenon is actually a symptom of a greater concern.[19] At issue is the coherence of the curriculum, or the degree to which mathematics topics are sequenced over the years. Perhaps equally important is the impact that instruction can have on learning. The National Research Council report *How Students Learn: Mathematics in the Classroom* cites the following problems:

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[17] Minn. Stat. § 120B.021, subd. 1(2).

[18] A copy of the proposed benchmarks that accompany the amended mathematics academics standards is attached to this SONAR.

[19] *See* http://www.ncee.org/acsd/newsletter/920.jsp. A biography for Schmidt can be found at http://ed-web2.educ.msu.edu/researchprofiles/search/profileview.asp?email=bschmidt@msu.edu.

- Instead of connecting with, building on, and refining the mathematical understandings, intuitions, and resourcefulness that students bring to the classroom, mathematics instruction often overrides students' reasoning processes, replacing them with a set of rules and procedures that disconnects problem solving from meaning-making.
- Instead of organizing the skills and competencies required to do mathematics fluently around a set of core mathematical concepts, those skills and competencies are often themselves the center, and sometimes the whole, of instruction.
- Because the acquisition of procedural knowledge is often divorced from meaning-making, students do not use metacognitive strategies when they engage in solving mathematical problems.[20]

The proposed revision of the Minnesota Academic Standards in Mathematics addresses this situation by providing standards and benchmarks that are more focused and coherent than those in current use, so that districts can more easily develop curriculum and instruction for their students. The standards proposed in this rulemaking process do not require specific instructional approaches. However, the developmental progression of knowledge and skills in the standards was informed by research on best practices in mathematics education.

To summarize the standards development sequence, the Committee initially focused on identifying the knowledge and skills needed for mastery of Algebra I by the end of Grade 8. Next, the Committee identified the prerequisite knowledge and skills for success in Algebra I as well as other mathematics content areas in the grade levels leading up to Grade 8. The committee then identified the knowledge and skills necessary for Algebra II in high school as well other content required for college and work readiness, thus building a coherent set of standards to improve K-12 mathematics education in Minnesota.

In order to identify the knowledge and skills needed, the standards committee consulted numerous documents in addition to the feedback provided by citizens online. Key documents included standards from other states and national standards frameworks as well as reports from government, business and educational organizations. The committee identified the kinds of knowledge and skills cited in these sources and analyzed the "gap" between the content of the national documents and the Minnesota standards. Recommendations from a number of expert reviewers of the draft revisions also were considered. In cases where opinions varied among reviewers, the Committee used the national documents to inform their decision.

Content Areas in the Standards and Key Documents Used to Develop Them

Minnesota's mathematics standards govern several broad content areas: number and operation, algebra, geometry and measurement, and data analysis and probability. At the various grade levels from Kindergarten through Grade 12, the mathematics standards are presented within these content areas. During the development process, the Committee made decisions about what content standards to include at each grade level in conjunction with recommendations from the previously mentioned experts as well as consultation with several highly regarded resources, including the following:

[20] National Research Council, *How Students Learn: Mathematics in the Classroom* 217 (2005).

- *Principles and Standards for School Mathematics*;
- *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics,* both from the National Council of Teachers of Mathematics (www.nctm.org);
- *College and Work Readiness Expectations,* written by the Minnesota P-16 Education Partnership working group;
- Standards found in the American Diploma Project of Achieve, Inc. (www.achieve.org); and
- *Recommended Standards for Information and Technology Literacy* from the Minnesota Educational Media Organization (MEMO) (www.memoweb.org).

The *Principles and Standards for School Mathematics*, created by the National Council of Teachers of Mathematics (NCTM) in 2000, identifies national standards and benchmarks in mathematics. States, districts, schools and teachers across the United States rely on this key document when developing standards and benchmarks. *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* is another NCTM resource that, like *Principles and Standards for School Mathematics*, is a highly respected resource that guides standards development efforts across the nation. *Curriculum Focal Points* identifies knowledge and skills that are essential for an educated citizenry and that provide the foundations for further mathematical learning. Specifically, the document delineates grade-by-grade developmental progression of mathematical knowledge.

College and Work Readiness Expectations identifies required skills and knowledge for successful entry into Minnesota colleges and the skilled workplace. The benchmarks from the American Diploma Project, developed by Achieve, Inc., are a compilation of expectations for college, work and citizenship readiness used in the development of mathematics standards in 30 states. Minnesota developed its *College and Work Readiness Expectations* in conjunction with its participation in a series of alignment institutes sponsored by the American Diploma Project.

A final document, *Recommended Standards for Information and Technology Literacy*, published by the Minnesota Educational Media Organization, describes the processes and skills a learner must understand and practice to meet a minimum level of information literacy. These five documents will be referred to repeatedly throughout this analysis in support of the proposed rules. Any conflicting recommendations from reviewers were resolved by referring to documents mentioned above.

The proposed rules are addressed individually in this SONAR. However, the Committee believed that it was important to emphasize the coherence of the standards as they develop from Kindergarten through Grade 12. Within each content area – and even between the main content areas – the standards are designed to build upon each other. Beginning in kindergarten, the standards generally emphasize and develop the content areas with the focus at each grade as follows:

Kindergarten: Counting and number relationships, with beginning work on combining and separating sets in preparation for adding and subtracting in first grade.

First grade: Building on kindergarten learning through concepts of adding and subtracting, and also learning about grouping tens and ones and composing and decomposing geometric shapes.

Second grade: Extending grouping to place value and addition and subtraction of basic facts and two-digit numbers.

Third grade: Multi-digit addition and subtraction as well as the development of strategies for multiplication and division.

Fourth grade: Multiplication and division facts, multi-digit multiplication, and work with fractions.

Fifth grade: Multi-digit division, addition and subtraction with fractions and decimals, and area.

Sixth grade: Ratios and probability, multiplication and division with positive rational numbers, surface area, and volume.

Seventh grade: The study of proportionality.

Eighth grade: The study of lines, linear equations, linear functions, and constant rates of change.

High school: The study of geometry, data analysis and probability, and algebra II.

The coherence of the standards as they develop from Kindergarten through Grade 12 is evident in the expansion of the number concepts in Kindergarten through Grade 5 and the shift from arithmetic to algebra in Grades 6 through 8. Within and between the main content areas, the standards are designed to build upon each other. Finally, in revising these mathematics standards as an integrated whole, the Committee considered current research regarding how students learn mathematics.

Developing Mathematical Proficiency

It is not enough to just introduce material at earlier grades; laying the groundwork for proficiency requires careful attention to the kinds of learning experiences that are provided for students and the learning processes required to succeed in mathematics learning. Though past practices placed an emphasis on procedural skills, this approach is no longer viable. It is now recognized that mathematical proficiency requires much more than procedural fluency. In its 2001 report *Adding It Up: Helping Children Learn Mathematics*, the National Research Council identifies five intertwined components of mathematical proficiency. These are: conceptual understanding, or comprehension of concepts, operations and relations; procedural fluency, which refers to skill in carrying out procedures flexibly, accurately, efficiently and appropriately; strategic competence, which involves the ability to formulate, represent and solve problems; adaptive reasoning, or the capacity for logical thought, reflection, explanation and justification;

and productive disposition, which involves the habitual inclination to see mathematics as sensible, useful and worthwhile as well as a belief in diligence and one's own efficacy.[21] In addition, the National Council of Teachers of Mathematics promotes the process standards – reasoning and proof, communication, connections, representations and problem solving – as integral to the acquisition as well as the application of the content standards. According to *Curriculum Focal Points,* "[w]ithout facility with these critical processes, a student's mathematical knowledge is likely to be fragile and limited in its usefulness."[22]

Furthermore, expert reviewers of the proposed rules urged that the mathematical reasoning subpart, which is included at Subpart 1 of the current rule for each grade level, should be integrated into specific content rule subparts. Steen stated that "the decision to separate reasoning ... from the content standards has the effect of marginalizing rather than strengthening their role in the standards."[23] Klein concurs stating, "[o]ne of the weaknesses of Minnesota's draft standards is the segregation of mathematical content and mathematical reasoning in different strands."[24] Klein explained that problem-solving is an indispensable part of learning mathematics and that mathematical reasoning should be an integral part of the content at all grade levels.[25] The Department agrees with this assessment and with the recommendations. Department program specialists and the Committee considered the recommendations balanced against observations of curriculum development under the existing standards used in Minnesota schools. Based on that work, the Department has found that reasoning is best considered when teaching content, rather than separated from the content. Therefore, the Department proposes to repeal the separate mathematical reasoning rule at each grade level and incorporate reasoning into the revised content standards and benchmarks to reflect this more integrated approach. In the proposed rules, reasoning standards are incorporated into the benchmarks of content standards for each major content area: number and operation; algebra; geometry and measurement; and data analysis and probability.

The rules will be analyzed by grade level and content area. The SONAR will begin with number and operation, building toward 8th grade algebra, geometry, and data analysis and probability for kindergarten through Grade 7. The 8th grade standards will be next, followed by the standards for high school students.

Standards for Kindergarten through Grade 7: Building to Algebra I in Grade 8

Central to the development of algebra at earlier grades – which is required by the legislative mandate of Algebra I in Grade 8 – is the notion that rather than cursory and repeated work, important ideas are focused on in more depth, allowing them to be developed more thoroughly. It is particularly difficult to separate the concepts of number and algebra in the elementary grades, so this next SONAR section will discuss both of these subparts together. Proficiency with

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[21] National Research Council, *Adding It Up: Helping Children Learn Mathematics* 116 (2001).
[22] National Council of Teachers of Mathematics, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* 5 (2006).
[23] Steen, L. (January 15, 2007). Review of Minnesota Academic Standards in Mathematics, p. 2.
[24] Klein, D. (January 10, 2007). Report on the Second Draft of the revised Minnesota K-12 Academic Standards in Mathematics, p. 2.
[25] Klein, D. (January 10, 2007). Report on the Second Draft of the revised Minnesota K-12 Academic Standards in Mathematics, p. 4.

numbers is seen as an appreciation for different notations of numbers, flexibility with translating from one representation to another, and connections among numbers and operations. Past results indicate that students have difficulty representing, connecting, and using numbers other than whole numbers.[26] The revised standards seek to provide requirements for learning with understanding so that various representations of numbers are equated, and so that situations in which they are used are recognized. Furthermore, as noted in the *Adding It Up: Helping Children Learn Mathematics* report, though the emphasis on "learning basic facts" in the United States has been on rote memorization of those facts, "research has shown that children benefit from learning experiences that acknowledge a progression, from using physical objects to represent situations on to using more sophisticated counting and reasoning strategies, such as deriving one number combination from another number combination (e.g., finding 7 + 8 by knowing that it is 1 more than 7 + 7 or, similarly, finding 7 × 6 as 7 more than 7 × 5)."[27] Increasingly sophisticated understanding and application of the properties are developed progressively at each grade level, with particular attention paid to learning issues surrounding equality throughout the work in the number standards and with numerical algorithms to facilitate achievement in algebra. As mentioned previously, mathematical proficiency is built on understanding and reasoning, and the committee tried to make this more evident in the revised standards that are proposed in these rules.

As students increase their facility with whole numbers, fractions are introduced. Learners require an understanding of fractions as numbers and, as with whole numbers, an emphasis on different notations and representations of fractions is essential. It is important to note that a clear understanding of how these fraction concepts relate to those of division, measurement, and ratio is also required for facility in operating with rational numbers. This is another example of the process standards, which focus on connections and representations that are essential for understanding mathematics. These process standards are addressed by reasoning benchmarks that support the proposed content standards. Research reveals that operations with fractions are challenging for students, and encourages states and other standards developers to incorporate approaches that build on student thinking in order to make operations with fractions more accessible and easier to master.[28] According to the *Adding It Up: Helping Children Learn Mathematics* report, contexts and use of objects to help students make sense of the operations offer more promise than rule-based approaches.[29]

Work with decimals requires a deeper focus on place value and understanding base-10 representations. Focusing on place value in Grade 2, and connecting it to arithmetic algorithms in all grades, will help to prepare students for work involving properties in algebra. The focus on ratios in Grade 6 sets the stage for proportionality in Grade 7, and then linear equations and linear functions in Grade 8. The *Adding It Up: Helping Children Learn Mathematics* report summarizes the difficulties in dealing with ratios and proportions:

> The concept of ratio is much more difficult than many people realize. *Proportional reasoning* is the term given to reasoning that involves the equality and manipulation of

[26] *Adding It Up: Helping Children Learn Mathematics,* p. 412.
[27] *Id.* at 413.
[28] *Id.* at 240.
[29] *Id.* at 416.

> ratios. Children often have difficulty comparing ratios and using them to solve problems. Many school mathematics programs fail to develop children's understanding of ratio comparisons and move directly to formal procedures for solving missing-value proportion problems. Research tracing the development of proportional reasoning shows that proficiency grows as students develop and connect different aspects of proportional reasoning. Further, the development of proportional reasoning can be supported by having students explore proportional situations in a variety of problem contexts using concrete materials or through data collection activities.[30]

The Committee balanced the recommendations from the NCTM *Curriculum Focal Points* with the goal of meeting the requirement of preparing all students for Algebra I in Grade 8. The Department believed that it was necessary to revise the academic standards for mathematics accordingly incorporates Algebra I in Grade 8 and that the NCTM *Curriculum Focal Points* provided reasonable recommendations. Hence, the proposed standards are necessary and reasonable. It is recommended that the 2003 standards be replaced with the proposed standards listed below. As discussed above, the Department proposes new number, algebra, geometry and data analysis standards that reflect not only the new Algebra I requirement for Grade 8, but also the recommendations of the Committee, the standards reviewers, and current thought in mathematics learning. The geometry and data analysis standards for Kindergarten through Grade 7 are discussed below the rule text.

3501.0700 KINDERGARTEN STANDARDS.

Subpart 1. **Number and operation**

A. The student will understand the relationship between quantities and whole numbers up to 31.

B. The student will use objects and pictures to represent situations involving combining and separating.

Subp. 2. **Algebra.** The student will recognize, create, complete, and extend patterns.

Subp. 3. **Geometry and measurement.**

A. The student will recognize and sort two- and three-dimensional shapes and use them to model real-world objects.

B. The student will compare and order objects according to location and measurable attributes.

3501.0705 GRADE 1 STANDARDS.

Subp. 1. **Number and operation.**

[30] National Research Council, *Adding It Up: Helping Children Learn Mathematics*, 417.

A. The student will count, compare, and represent whole numbers up to 120, with an emphasis on groups of tens and ones.

B. The student will use a variety of models and strategies to solve addition and subtraction problems in real-world and mathematical contexts.

Subp. 2. **Algebra.**

A. The student will recognize and create patterns and use rules to describe patterns.

B. The student will use number sentences involving addition and subtraction basic facts to represent and solve real-world and mathematical problems and the student will create real-world situations corresponding to number sentences.

Subp. 3. **Geometry and measurement.**

A. The student will describe characteristics of basic shapes, and use basic shapes to compose and decompose other objects in various contexts.

B. The student will use basic concepts of measurement in real-world and mathematical situations involving length, time, and money.

3501.0710 GRADE 2 STANDARDS.

Subp. 1. **Number and operation.**

A. The student will compare and represent numbers up to 1,000 with an emphasis on place value and equality.

B. The student will demonstrate mastery of addition and subtraction basic facts. The student will add and subtract one- and two-digit numbers in real-world and mathematical problems.

Subp. 2. **Algebra.**

A. The student will recognize, create, describe and use patterns and rules to solve real-world and mathematical problems.

B. The student will use number sentences involving addition, subtraction and unknowns to represent and solve real-world and mathematical problems. The student will create real-world situations corresponding to number sentences.

Subp. 3. **Geometry and measurement.**

A. The student will identify, describe and compare basic shapes according to their geometric attributes.

B. The student will understand length as a measurable attribute and the student will use tools to measure length.

C. The student will use time and money in real-world and mathematical situations.

3501.0715 GRADE 3 STANDARDS.

Subp. 1. **Number sense and operation.**

A. The student will compare and represent whole numbers up to 100,000 with an emphasis on place value and equality.

B. The student will add and subtract multidigit whole numbers. The student will represent multiplication and division in various ways and solve real-word and mathematical situations.

C. The student will understand meanings and uses of fractions in real-world and mathematical situations.

Subp. 2. **Algebra.**

A. The student will use single-operation input-output rules to represent patterns and relationships and to solve real-world and mathematical problems.

B. The student will use number sentences involving multiplication and division basic facts and unknowns to represent and solve real-world and mathematical problems. The student will create real-world situations corresponding to number sentences.

Subp. 3. **Geometry and measurement.**

A. The student will use geometric attributes to describe and create shapes in various contexts.

B. The student will understand perimeter as a measurable attribute of real-world and mathematical objects. The student will use various tools to measure distances.

C. The student will use time, money, and temperature to solve real-world and mathematical problems.

Subp. 4. **Data analysis.** The student will collect, organize, display and interpret data. The student will use labels and a variety of scales and units in displays.

3501.0720 GRADE 4 STANDARDS.

Subp. 1. **Number and operation.**

A. The student will demonstrate mastery of multiplication and division basic facts. The student will multiply multidigit numbers and solve real-world and mathematical problems using arithmetic.

B. The student will represent and compare fractions and decimals in real-world and mathematical situations. The student will use place value to understand how decimals represent quantities.

Subp. 2. **Algebra.**

A. The student will use input-output rules, tables, and charts to represent patterns and relationships and to solve real-world and mathematical problems.

B. The student will use number sentences involving multiplication, division and unknowns to represent and solve real-world and mathematical problems. The student will create real-world situations corresponding to number sentences.

Subp. 3. **Geometry and measurement.**

A. The student will name, describe, classify and sketch polygons.

B. The student will understand angle and area as measurable attributes of real-world and mathematical objects. The will use various tools to measure angles and areas.

C. The student will use translations, reflections, and rotations to establish congruency and understand symmetries.

Subp. 4. **Data analysis.** The student will collect, organize, display, and interpret data, including data collected over a period of time and data represented by fractions and decimals.

3501.0725 GRADE 5 STANDARDS.

Subp. 1. **Number and operation.**

A. The student will divide multidigit numbers. The student will solve real-world and mathematical problems using arithmetic.

B. The student will read, write, represent and compare fractions and decimals. The student will recognize and write equivalent fractions and convert between fractions and decimals. The student will use fractions and decimals in real-world and mathematical situations.

C. The student will add and subtract fractions, mixed numbers, and decimals to solve real-world and mathematical problems.

Subp. 2. **Algebra.**

A. The student will recognize and represent patterns of change. The student will use patterns, tables, graphs and rules to solve real-world and mathematical problems.

B. The student will use properties of arithmetic to generate equivalent numerical expressions and evaluate expressions involving whole numbers.

C. The student will understand and interpret equations and inequalities involving variables and whole numbers and use them to represent and solve real-world and mathematical problems.

Subp. 3. **Geometry and measurement.**

A. The student will describe, classify and draw representations of three-dimensional figures.

B. The student will determine the area of triangles and quadrilaterals. The student will determine the surface area and volume of rectangular prisms in various contexts.

Subp. 4. **Data analysis.**The student will display and interpret data and determine mean, median and range.

3501.0730 GRADE 6 STANDARDS.

Subp. 1. **Number and operation.**

A. The student will read, write, represent and compare positive rational numbers expressed as fractions, decimals, percents, and ratios and write positive integers as products of factors. The student will use these representations in real-world and mathematical situations.

B. The student will understand the concept of ratio and its relationship to fractions and to the multiplication and division of whole numbers. The student will use ratios to solve real-world and mathematical problems.

C. The student will multiply and divide decimals, fractions and mixed numbers. The student will solve real-world and mathematical problems using arithmetic with positive rational numbers.

Subp. 2. **Algebra.**

A. The student will recognize and represent relationships between varying quantities. The student will translate from one representation to another. The student will use patterns, tables, graphs and rules to solve real-world and mathematical problems.

B. The student will use properties of arithmetic to generate equivalent numerical expressions and evaluate expressions involving positive rational numbers.

C. The student will understand and interpret equations and inequalities involving variables and positive rational numbers. The student will use equations and inequalities to represent real-world and mathematical problems. The student will use the idea of maintaining equality to solve equations. The student will interpret solutions in the original context.

Subp. 3. **Geometry and measurement.**

A. The student will calculate perimeter, area, surface area and volume of two-and three-dimensional figures to solve real-world and mathematical problems.

B. The student will understand and use relationships between angles in geometric figures.

C. The student will choose appropriate units of measurement and use ratios to convert within measurement systems to solve real-world and mathematical problems.

Subp. 4. **Data analysis and probability.** The student will use probabilities to solve real-world and mathematical problems. The student will represent probabilities using fractions, decimals and percents.

3501.0735 GRADE 7 STANDARDS.

Subp. 1. **Number and operation.**

A. The student will apply, read, write, represent and compare positive and negative rational numbers, expressed as integers, fractions and decimals.

B. The student will calculate with positive and negative rational numbers and rational numbers with whole number exponents to solve real-world and mathematical problems.

Subp. 2. **Algebra.**

A. The student will understand the concept of proportionality in real-world mathematical situations, and distinguish between proportional and other relationships.

B. The student will recognize proportional relationships in real-world and mathematical situations. The student will represent these and other relationships with tables, verbal descriptions, symbols and graphs. The student will solve problems involving proportional relationships and explain results in the original context.

C. The student will apply understanding of order of operations and algebraic properties to generate equivalent numerical and algebraic expressions containing positive and negative rational numbers and grouping symbols. The student will evaluate such expressions.

D. The student will represent real-world and mathematical situations using equations with variables. The student will solve equations symbolically, using the properties of equality. The student will also solve equations graphically and numerically. The student will interpret solutions in the original context.

Subp. 3. **Geometry and measurement.**

A. The student will use reasoning with proportions and ratios to determine measurements, justify formulas, and solve real-world and mathematical problems involving circles and related geometric figures.

B. The student will analyze the effect of change of scale, translations and reflections on the attributes of two-dimensional figures.

Subp. 4. **Data analysis and probability.**

A. The student will use mean, median, and range to draw conclusions about data and make predictions.

B. The student will display and interpret data in a variety of ways, including circle graphs and histograms.

C. The student will calculate probabilities and reason about probabilities using proportions to solve real-world and mathematical problems.

Geometry Standards for K-7

The geometry standards were developed together with all of the other mathematics standards contained in this proposed mathematics standards package, and should be considered a natural part of the Committee's attempt to develop a cohesive set of standards that work as a whole between content areas and grade levels. The discussion found in the introductory Rule-by-Rule Analysis sections contains much of the background and reasoning behind the development of these proposed geometry standards.

Following recommendations from resources including Schmidt, NCTM's *Curriculum Focal Points*, and Achieve, Inc., in the early grades the geometry content is used to support the development of number. The committee tried to adhere to recommendations to create a standards document that provided much more focus and attention to fundamental ideas. Particular attention was paid to recommendations to eliminate repetition in the standards. In addition, placement of particular topics was guided by the NCTM *Curriculum Focal Points.*

Data Analysis and Probability Standards for K-7

The development of these data analysis and probability standards was a natural outgrowth of the Committee's intent to focus on the mathematics standards as a whole, and to develop standards that would address global issues, such as coherence between content areas and between grade levels. Thus, it is difficult to separate the explanation of these specific standards from the overall discussion of the Committee's work on these proposed mathematics standards in their entirety. The discussion found in the introductory Rule-by-Rule Analysis sections contains much of the background and reasoning behind the development of these proposed data analysis and probability standards.

The committee followed recommendations from NCTM's *Curriculum Focal Points* that were discussed above, as well as the national reviewers of the proposed standards. The Department determined that these recommendations by the Committee would provide for a more focused set of standards that specifies mastery of data and probability standards at later years, while also providing for more focus on the number standards in earlier years. Thus, the rules governing data analysis and probability standards appear in Grade 3 through Grade 12.

Standards for Grade 8: Establishing Algebra I in Grade 8

The next section of this document will provide justification for how the rules were revised to meet the requirements for Algebra I in Grade 8. Putting Algebra I in the Grade 8 standards results in revised standards at the secondary level that emphasize higher mathematics proficiency so that all Minnesota students leave secondary school ready for college and work. It also shapes the mathematics standards in each of the elementary grades, because students must achieve

certain proficiencies in each of the mathematics content areas in order to be prepared to learn Algebra I by the end of Grade 8. At the same time, the standards revision committee considered how the elementary standards could better reflect new research on student learning in order to better prepare students to learn Algebra I in Grade 8, and to learn higher mathematics in the secondary grades. Thus, the Committee worked forwards and backwards from Grade 8 as it revised the standards to reflect a legislative mandate that Minnesota students be better prepared for college and work. This organizational structure for the SONAR provides a glimpse at the process used by the standards revision committee.

In its *Adding It Up: Helping Children Learn Mathematics* report, the National Research Council states that although research regarding the transition from arithmetic to algebra has received extensive attention in recent years, more research is required to fully understand how students develop awareness of the use and appreciation of algebraic expressions.[31] However, it is now widely accepted that the study of algebra does not have to begin with a formal course in high school, as has been the tradition. Rather, research focuses on ways that elementary and middle school curricula can be used to support the development of algebraic reasoning over many grades. Hence, rules for standards at earlier grades are written to support the learning of Algebra I by the end of Grade 8, where the focus is on constant rates of change and mastery of linear equations and functions.

3501.0740 GRADE 8 STANDARDS.

Subp. 1. **Number and operation.** The student will read, write, compare, classify and represent real numbers, and use them to solve problems in various contexts.

Subp. 2. **Algebra.**

A. The student will understand the concept of function in real-world and mathematical situations and distinguish between linear and nonlinear functions.

B. The student will recognize linear functions in real-world and mathematical situations and represent linear functions and other functions with tables, verbal descriptions, symbols and graphs. The student will solve problems involving these functions and explain results in the original context.

C. The student will generate equivalent numerical and algebraic expressions and use algebraic properties to evaluate expressions.

[31] National Research Council, Adding It Up: Helping Children Learn Mathematics 279, 419.

D. The student will represent real-world and mathematical situations using equations and inequalities involving linear expressions. The student will solve equations and inequalities symbolically and graphically. The student will interpret solutions in the original context.

Subp. 3. **Geometry and measurement.**

A. The student will solve problems involving right triangles using the Pythagorean Theorem and its converse.

B. The student will solve problems involving parallel and perpendicular lines on a coordinate system.

Subp. 4. **Data analysis and probability.** The student will interpret data using scatterplots and approximate lines of best fit. The student will use lines of best fit to draw conclusions about data.

The national reviewers of the Committee's work support the development of algebra as presented in these revised standards. For example, Leinwand, a reviewer of the proposed standards, writes, "You are to be commended for making what I believe are correct and rational decisions about what elements of algebra you include in Grade 8."[32] The rules for Grade 8 are clearly focused on linear functions. Careful attention was given to including content in the other subparts to support these new expectations. These changes were necessary to provide clear standards for Algebra I in Grade 8. As discussed in the next section, the content of the Grade 8 standards found in the current rule are placed in standards (rule) or benchmarks in the earlier grades to assure that students are prepared for and have adequate time to master Algebra I in Grade 8.

Because standards identify content that is to be mastered, the Committee began by identifying content that is typically taught in an Algebra I course, but is generally not mastered until Algebra II. For example, though content involving quadratic and exponential relationships may be taught in an Algebra I course, in many cases, this content is taught again in an Algebra II course. Furthermore, many students continue to struggle with the understanding of linear equations and functions. In an effort to provide a more coherent set of standards, it was determined that all students should master content involving linear relationships in Algebra I.

High School Standards

The proposed high school algebra standards were revised to reflect changes resulting from moving Algebra I content to Grade 8. The proposed standards are aligned to Minnesota's

[32] Leinwand, S, (January 16, 2007). Memorandum regarding review of revised Minnesota K-12 Academic Standards in Mathematics, p. 2.

College and Work Readiness Expectations as well as to the benchmarks from the American Diploma Project, developed by Achieve, Inc. Mastery of Algebra I by the end of Grade 8 provides more time for all high school students to master the high school standards, including the standards for Algebra II. This clear delineation between Algebra I and high school mathematics is designed to help districts provide more focused and intentional learning experiences so that all students are successful in Algebra II.

3501.0745 GRADES 9 THROUGH 11 STANDARDS.

Subp. 1. **Algebra.**

A. The student will recognize the concept of function and identify important features of functions and other relations using symbolic and graphical methods where appropriate.

B. The student will recognize linear, quadratic, exponential, and other common functions in real-world and mathematical situations. The student will represent these functions with tables, verbal descriptions, symbols, and graphs and solve problems involving these functions, and explain results in original context.

C. The student will generate equivalent algebraic expressions involving polynomials and radicals and use algebraic properties to evaluate expressions.

D. The student will represent real-world and mathematical situations using equations and inequalities involving linear, quadratic, exponential, and nth root functions. The student will interpret solutions in the original context.

Subp. 2. **Geometry and measurement.**

A. The student will calculate measurements of plane and solid geometric figures. The student will know that physical measurements depend on the choice of a unit and that they are approximates.

B. The student will construct logical arguments, based on axioms, definitions, and theorems, to prove theorems and other results in geometry.

C. The student will know and apply properties of geometric figures to solve real-world and mathematical problems and to logically justify results in geometry.

D. The student will solve real-world and mathematical geometric problems using algebraic methods.

Subp. 3. **Data analysis and probability.**

A. The student will display and analyze data. The student will use various measures associated with data to draw conclusions, identify trends and describe relationships.

B. The student will explain the uses of data and statistical thinking to draw inferences, make predictions, and justify conclusions.

C. The student will calculate probabilities and apply probability concepts to solve real-world and mathematical problems.

The high school geometry standards were developed together with all of the other mathematics standards contained in this proposed mathematics standards package, and should be considered a natural part of the committee's attempt to develop a cohesive set of standards that work as a whole between content areas and grade levels. The discussion found in the introductory Rule-by-Rule Analysis sections contains much of the background and reasoning behind the development of these proposed geometry standards. The rigor of the standards at the high school level has been increased in asking that students logically justify results and prove theorems in geometry.

The rules governing data analysis and probability standards appear in Grade 3 through Grade 12. A major portion of the data analysis and probability content is located at the high school level, where students' facility with numbers and randomness provides a readiness for the data analysis and probability standards. The proposed standards in data analysis and probability for high school follow the recommendations of Achieve, Inc. for college and work readiness. Changes include a greater focus on using statistical thinking to interpret phenomena as represented in numerical data. The Department agrees with the committee's recommendations.

The Department agrees with the recommendations from the committee and recommends the replacement of the 2003 rules regarding academic standards for mathematics with the standards provided in these proposed rules.

Conclusion

Based on the foregoing, the proposed rules are both needed and reasonable.

6/5/08
Date

Chas Anderson, Deputy Commissioner

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