

# Minnesota Academic Standards

## Mathematics K-12

May 19, 2003



Minnesota Department of Education

## ORGANIZATION OF THIS DOCUMENT

The Minnesota Academic Standards in Mathematics are organized by grade level into five general *strands*: 1) Mathematical Reasoning, 2) Number Sense, Computation, and Operations, 3) Patterns, Functions, and Algebra, 4) Data Analysis, Statistics, and Probability, and 5) Spatial Sense, Geometry, and Measurement. Two or more categories, or *sub-strands* of standards, are listed under four of the five strands. For example, the Patterns, Functions, and Algebra strand includes the sub-strands of Patterns, Functions, and Algebra (Algebraic Thinking).

The strands are identified by Roman numerals (I, II, III, IV, V) and the sub-strands by upper case letters (A, B, C, etc.). Each sub-strand is clarified with a standard statement. The benchmarks are identified by ordinal numbers (1, 2, 3, etc.). The example below illustrates the relationship between a strand, sub-strand, standard, and benchmark.

This document is grounded in the belief that all students should learn important mathematical concepts, skills, and relationships with understanding. The five strands presented here describe a connected body of mathematical knowledge. The standards are placed at the grade level where mastery is expected. Mastery of these standards will require preliminary work at earlier grades.

### Example

KINDERGARTEN	
<i>Strand</i>	<b>I. PATTERNS, FUNCTIONS, AND ALGEBRA</b>
<i>Sub-strand</i>	<b>A. Patterns and Functions</b>
<i>Standards</i>	Standard: Sort, classify and compare objects based on their attributes. Understand simple repeating patterns.
<i>Benchmarks</i>	<ol style="list-style-type: none"><li>1. Sort objects in a set by one attribute such as size, shape, color or thickness.</li><li>2. Identify an object that does not belong in a set.</li><li>3. Recognize, describe, and extend patterns involving up to three elements using objects, pictures, sounds, or movements.</li></ol>

The next page outlines the organization of the Mathematics standards for all grade levels.

## FRAMEWORK FOR THE K-12 MATHEMATICS STANDARDS

### GRADE LEVEL (Kindergarten, Grade 1, Grade 2, etc.)

#### I. MATHEMATICAL REASONING

Standard statement

1. Benchmark statement
2. Benchmark statement, etc.

#### II. NUMBER SENSE, COMPUTATION, AND OPERATIONS

##### A. Number Sense

Standard statement

1. Benchmark statement
2. Benchmark statement, etc.

##### B. Computation and Operation

Standard statement

1. Benchmark statement
2. Benchmark statement, etc.

#### III. PATTERNS, FUNCTIONS, AND ALGEBRA

##### A. Patterns and Functions

Standard statement

1. Benchmark statement
2. Benchmark statement, etc.

##### B. Algebra (Algebraic Thinking)

Standard statement

1. Benchmark statement
2. Benchmark statement, etc.

#### IV. DATA ANALYSIS, STATISTICS, AND PROBABILITY

##### A. Data and Statistics

Standard statement

1. Benchmark statement
2. Benchmark statement, etc.

##### B. Probability

Standard statement

1. Benchmark statement
2. Benchmark statement, etc.

#### V. SPATIAL SENSE, GEOMETRY, AND MEASUREMENT

##### A. Spatial Sense

Standard statement

1. Benchmark statement
2. Benchmark statement, etc.

##### B. Geometry

Standard statement

1. Benchmark statement
2. Benchmark statement, etc.

##### C. Measurement

Standard statement

1. Benchmark statement
2. Benchmark statement, etc.

## KINDERGARTEN

These standards and benchmarks are to be used as a source document for identifying what all students should know and be able to do to demonstrate mathematical proficiency. To determine grade level placement of specific standards and benchmarks, judgment by experienced teachers was used to determine at what grade level 80% of children would master the specific material.

The current document identifies the grade at which mastery of each concept is expected but does not identify when those concepts are introduced and reinforced. Schools must determine where in their curriculum these concepts would be introduced and reinforced so that they may be assessed at the indicated grade level.

Teachers must develop and enrich students' knowledge of mathematics beyond what is outlined in this document. It is critical for teachers to recognize the entire progression of standards and benchmarks before and after their grade level.

*The grade level designations in the K-2 Minnesota Academic Standards are strongly recommended. However, school districts may place the K-2 standards at grade levels that accommodate their particular curriculum, provided that all standards have been mastered by the end of grade 2.*

### I. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining four content strands.

*Reasoning lies at the heart of mathematical proficiency. As such, it is an essential strand of mathematics. The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.*

The student will:

1. Create and solve word problems using actions, objects, words, pictures, or numbers.
2. Estimate and check that answers are reasonable.
3. Explain to others how a problem was solved.

### II. NUMBER SENSE, COMPUTATION, AND OPERATIONS

#### A. Number Sense

Standard: Represent quantities using whole numbers and understand relationships among whole numbers.

The student will:

1. Count forward to 31, backward from 10.
2. Count the number of objects in a set and identify the quantity.
3. Compare the number of objects in two or more sets.
4. Given a number, identify one more or one less.

### **B. Computation and Operation**

Standard: Add and subtract whole numbers up to 6 in real-world and mathematical problems.

The student will:

1. Recognize the number of objects up to 6, without counting.
2. Add and subtract whole numbers up to 6, using concrete objects.

## **III. PATTERNS, FUNCTIONS AND ALGEBRA**

### **A. Patterns and Functions**

Standard: Sort, classify and compare objects based on their attributes. Understand simple repeating patterns.

The student will:

1. Sort objects in a set by one attribute such as size, shape, color or thickness.
2. Identify an object that does not belong in a set.
3. Recognize, describe and extend repeating patterns involving up to three elements using objects, pictures, sounds or movements.

### **B. Algebra (Algebraic Thinking)**

(Standards under this heading may be locally determined.)

## **IV. DATA ANALYSIS, STATISTICS AND PROBABILITY**

### **A. Data and Statistics**

Standard: Depict data with objects and pictures.

The student will:

1. Represent data about classmates or their surroundings by using objects or pictures.

### **B. Probability**

(Standards under this heading may be locally determined.)

## **V. SPATIAL SENSE, GEOMETRY, AND MEASUREMENT**

### **A. Spatial Sense**

Standard: Understand meaning of terms used to describe location and placement of objects.

The student will:

1. Locate and describe placement of objects with terms such as: on, inside, outside, above, below, over, under, beside, between, in front of, behind, next to, top, bottom.

### **B. Geometry**

Standard: Sort two- and three-dimensional shapes.

The student will:

1. Sort two- and three-dimensional shapes according to their geometrical attributes.

### **C. Measurement**

Standard: Understand terms and comparative language used in various measurement situations. Identify tools to measure time. Identify coins.

The student will:

1. Compare and order objects by length, weight, volume, temperature or size and use appropriate vocabulary such as longer than, holds more, smaller.
2. Know that clocks and calendars are instruments to measure time.
3. Recognize the following coins: penny, nickel, dime and quarter.
4. Compare and order events based on time and use appropriate vocabulary such as yesterday, today or tomorrow to describe relative time.

## GRADE 1

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### I. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining four content strands.

*Note about assessment of this standard: The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.*

The student will:

1. Create and solve word problems using actions, objects, words, pictures or numbers.
2. Estimate and check that answers are reasonable.
3. Explain to others how a problem was solved.

### II. NUMBER SENSE, COMPUTATION AND OPERATIONS

#### A. Number Sense

Standard: Understand place value, ways of representing whole numbers and relationships among whole numbers. Understand the concept of one half.

The student will:

1. Read, write numerals for, compare and order numbers to 120.
2. Count by 2s to 30 and by 5s to 120.
3. Count backwards from 30.
4. Demonstrate understanding of odd and even quantities up to 12.
5. Represent whole numbers up to 20 in various ways, maintaining equality.
6. Identify one half of a set of concrete objects.

### **B. Computation and Operation**

Standard: Add and subtract one-digit whole numbers in real-world and mathematical problems.

The student will:

1. Use one-digit addition and subtraction to solve real-world and mathematical problems.
2. Find the sum of three one-digit numbers.

## **III. PATTERNS, FUNCTIONS AND ALGEBRA**

### **A. Patterns and Functions**

Standard: Sort, classify and compare objects based on their attributes. Understand repeating patterns.

The student will:

1. Sort, classify, and compare objects in a set in more than one way.
2. Recognize, describe, and extend repeating patterns involving up to four elements.

### **B. Algebra (Algebraic Thinking)**

(Standards under this heading may be locally determined.)

## **IV. DATA ANALYSIS, STATISTICS AND PROBABILITY**

### **A. Data and Statistics**



Standard: Gather and record data in real-world and mathematical problems.

The student will:

1. Gather and record data about classmates and their surroundings in a simple graph.
2. Identify patterns in simple graphs.

### **B. Probability**

(Standards under this heading may be locally determined.)

## **V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT**

### **A. Spatial Sense**

Standard: Explore the concept of symmetry in real-world situations.

The student will:

1. Explore symmetry of objects and designs through mirrors or paper folding.

### **B. Geometry**

Standard: Use attributes of two- and three-dimensional shapes to identify them and distinguish between them.

The student will:

1. Sort and describe two- and three-dimensional shapes according to their geometrical attributes.

### **C. Measurement**

Standard: Measure length, time, and money using appropriate tools or units to solve real-world and mathematical problems.

The student will:

1. Estimate and measure length and capacity using non-standard units.
2. Tell time to hour and half-hour on analog and digital clocks.

3. Using a calendar, identify the date, day of the week, month, year, yesterday, today and tomorrow.
4. Combine pennies, nickels or dimes to equal one dollar.

## GRADE 2

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### I. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining four content strands.

*Note about assessment of this standard: The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.*

The student will:

1. Create and solve word problems using actions, objects, words, pictures or numbers.
2. Estimate and check that answers are reasonable.
3. Explain to others how a problem was solved.

### II. NUMBER SENSE, COMPUTATION AND OPERATIONS

#### A. Number Sense

Standard: Understand place value, ways of representing whole numbers and relationships among whole numbers. Understand the concept of unit fractions.

The student will:

1. Read, write with numerals, compare and order numbers to 999.
2. Count by 2s, 5s, 10s from any given whole number.
3. Understand and demonstrate the significance of groups of 10 in the base 10 number system.
4. Represent numbers in equivalent ways.
5. Recognize, name, compare and represent unit fractions with drawings or concrete materials.

### **B. Computation and Operation**

Standard: Compute fluently and make reasonable estimates with whole numbers in real-world and mathematical problems.

The student will:

1. Use one- and two-digit addition and subtraction to solve real-world and mathematical problems.
2. Demonstrate understanding of the relationships between odd and even numbers in addition and subtraction such as,  $\text{odd} + \text{odd} = \text{even}$  or  $\text{odd} - \text{even} = \text{odd}$ .
3. Understand the concept of multiplication as repeated addition or in rectangular arrays.
4. Understand the concept of division as repeated subtraction or sharing equally.

## **III. PATTERNS, FUNCTIONS AND ALGEBRA**

### **A. Patterns and Functions**

Standard: Understand repeating, growing and shrinking patterns.

The student will:

1. Recognize, create and extend repeating, growing and shrinking patterns using numbers, concrete objects and pictures.

### **B. Algebra (Algebraic Thinking)**

Standard: Understand basic properties of addition and subtraction.

The student will:

1. Describe what happens when zero is added to a number or subtracted from a number.
2. Generate equivalent expressions for a given number such as  $24 = 17 + 7$  or  $24 = 100 - 76$ .
3. Determine the truth-value of an equation such as: true or false?  $7 = 5 + 1$ .
4. Understand that adding two numbers in any order results in the same sum.
5. Understand that grouping numbers in multiple addend problems, in any order, results in the same sum.

#### **IV. DATA ANALYSIS, STATISTICS AND PROBABILITY**

##### **A. Data and Statistics**

Standard: Collect and represent data in real-world and mathematical problems.

The student will:

1. Collect and record categorical data.
2. Create pictographs and real-object graphs to represent data.
3. Identify patterns in graphs or data sets.

##### **B. Probability**

(Standards under this heading may be locally determined.)

#### **V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT**

##### **A. Spatial Sense**

Standard: Understand the concept of symmetry and apply to simple drawings.

The student will:

1. Create symmetrical patterns and designs.

##### **B. Geometry**

Standard: Use attributes of two- and three-dimensional shapes to identify them and distinguish between them.

The student will:

1. Investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.
2. Sort, classify, compare and describe two- and three-dimensional objects according to their geometrical attributes.

### **C. Measurement**

Standard: Measure length, time, temperature and money using appropriate tools and units to solve real-world and mathematical problems.

The student will:

1. Estimate standard and nonstandard linear measurements, then measure to check answer.
2. Tell time to the quarter hour, half hour and hour using analog and digital clocks, distinguishing between a.m. and p.m.
3. Know relationships among units of time such as minutes in an hour, days in a month and weeks in a year.
4. Read and write amounts of money using \$ for dollar, ¢ for cents, and proper placement of the decimal point with amounts of money.
5. Combine coins to create amounts up to one dollar.

## GRADE 3

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### I. MATHEMATICAL REASONING

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*Note about assessment of this standard: The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.*

The student will:

1. Communicate, reason and represent situations mathematically.
2. Solve problems by distinguishing relevant from irrelevant information, sequencing and prioritizing information and breaking multi-step problems into simpler parts.
3. Evaluate the reasonableness of the solution by considering appropriate estimates and the context of the original problem.
4. Know when it is appropriate to estimate and when an exact answer with whole numbers, fractions or decimals is needed.
5. Express a written problem in suitable mathematical language, solve the problem and interpret the result in the original context.
6. Support mathematical results using pictures, numbers and words to explain why the steps in a solution are valid and why a particular solution method is appropriate.

## II. NUMBER SENSE, COMPUTATION AND OPERATIONS

### A. Number Sense

Standard: Represent whole numbers in various ways to quantify information and to solve real-world and mathematical problems. Understand the concept of decimals and common fractions.

The student will:

1. Read, write with numerals, compare and order whole numbers to 9,999.
2. Represent up to 4-digit whole numbers in various ways maintaining equivalence, such as  $3206 = (32 \times 100) + 6$  or  $3206 = 3200 + 6$ .
3. Know how fractions are related to the whole, such as four-fourths equal a whole or three fourths equal three of four equal parts of a whole.
4. Represent and write fractions with pictures, models and numbers.

### B. Computation and Operation

Standard: Compute fluently and make reasonable estimates with whole numbers in real-world and mathematical problems. Understand addition and subtraction and how they relate to one another. Understand the concepts of multiplication and division.

The student will:

1. Use addition of up to three whole number addends, containing up to four digits each in real-world and mathematical problems.
2. Use subtraction with up to three digit whole numbers in real-world and mathematical problems.
3. Use the inverse relationship of addition and subtraction to compute and check results.
4. Demonstrate mastery of basic addition facts for addends 0 through 9, without a calculator.
5. Demonstrate mastery of subtraction facts that are inverses of the basic addition facts, without a calculator.
6. Demonstrate an understanding of the multiplication facts through 10 using concrete models.



7. Use models to solve multiplication and division problems and use number sentences to record the solutions.

### III. PATTERNS, FUNCTIONS AND ALGEBRA

#### A. Patterns and Functions

Standard: Understand and describe patterns in numbers and shapes.

The student will:

1. Create and identify patterns in numbers and shapes and explain how to extend those patterns.

#### B. Algebra (Algebraic Thinking)

Standard: Add and subtract whole numbers in the correct order to solve real-world and mathematical problems.

The student will:

1. Identify a missing number or operation in a simple arithmetic equation such as  $3 \_ 4 = 7$  or  $9 - \_ = 2$ .
2. Use the properties of addition and subtraction that involve ordering, grouping and the number 0, to do simple computations with whole numbers.

### IV. DATA ANALYSIS, STATISTICS AND PROBABILITY

#### A. Data and Statistics

Standard: Represent and interpret data in real-world and mathematical problems.

The student will:

1. Read and interpret data from circle graphs using halves, thirds and quarters.
2. Collect data using observations or surveys and represent the data with pictographs and line plots with appropriate title and key.

#### B. Probability

Standard: Explore the basic concept of probability.

**V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT****A. Spatial Sense**

Standard: Understand the concept of reflection symmetry as applied to geometric shapes. Understand how representations of shapes are affected by various motions.

The student will:

1. Identify lines of symmetry in geometric shapes.
2. Recognize and predict the position and orientation of a shape after a single flip, slide or turn.

**B. Geometry**

Standard: Classify shapes by specified attributes. Identify simple shapes within complex shapes.

The student will:

1. Identify, describe and classify two-dimensional shapes according to number and length of sides and kinds of angles.
2. Identify common two- and three-dimensional shapes that are components of more complex shapes.

**C. Measurement**

Standard: Measure and calculate length, time, weight, temperature and money using appropriate tools and units to solve real-world and mathematical problems.

The student will:

1. Select an appropriate tool and identify the appropriate unit to measure time, length, weight and temperature.
2. Find the perimeter of a polygon with whole number sides.
3. Know relationships between units of length in a system of measurement, such as 12 inches equals 1 foot or 100 centimeters equals 1 meter.
4. Tell time to the minute using digital and analog time.
5. Determine elapsed time to the minute.
6. Make change using as few coins as possible up to a dollar.

## GRADE 4

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### I. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining four content strands.

*Note about assessment of this standard: The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.*

The student will:

1. Communicate, reason and represent situations mathematically.
2. Solve problems by distinguishing relevant from irrelevant information, sequencing and prioritizing information and breaking multi-step problems into simpler parts.
3. Evaluate the reasonableness of the solution by considering appropriate estimates and the context of the original problem.
4. Know when it is appropriate to estimate and when an exact answer with whole numbers, fractions or decimals is needed.
5. Express a written problem in suitable mathematical language, solve the problem and interpret the result in the original context.
6. Support mathematical results using pictures, numbers, and words to explain why the steps in a solution are valid and why a particular solution method is appropriate.

## II. NUMBER SENSE, COMPUTATION AND OPERATIONS

### A. Number Sense

Standard: Represent whole numbers in various ways to quantify information and to solve real-world and mathematical problems. Understand the concept of fractions and decimals.

The student will:

1. Read and write whole numbers to 100,000, in numerals and words.
2. Compare and order whole numbers.
3. Use fractions and decimals to solve problems representing parts of a whole, parts of a set and division of whole numbers by whole numbers in real-world and mathematical problems.
4. Use rounding and estimation with whole numbers to solve real-world and mathematical problems.

### B. Computation and Operation

Standard: Compute fluently and make reasonable estimates with whole numbers in real-world and mathematical problems. Understand the meanings of arithmetic operations and how they relate to one another.

The student will:

1. Use addition and subtraction of multi-digit whole numbers to solve multi-step real-world and mathematical problems.
2. Add up to three whole numbers containing up to three digits each, without a calculator.
3. Subtract whole numbers containing up to three digits each, without a calculator.
4. Demonstrate mastery of multiplication facts for the numbers 0-10, without a calculator.
5. Use multiplication and division of whole numbers to solve simple real-world and mathematical problems.
6. Use the inverse relationship of multiplication and division to compute and check results.

7. Multiply single digit multiples of powers of ten such as  $300 \times 60$  or  $70 \times 3$ , mentally.

### III. PATTERNS, FUNCTIONS AND ALGEBRA

#### A. Patterns and Functions

Standard: Understand and describe patterns in tables and graphs.

The student will:

1. Examine and describe patterns in tables and graphs.

#### B. Algebra (Algebraic Thinking)

Standard: Apply arithmetic operations in the correct order to compute with whole numbers in real-world and mathematical problems.

The student will:

1. Identify a missing number or operation in a simple arithmetic equation such as  $3 \_ 4 = 12$  or  $45 / \_ = 9$ .
2. Use the properties of arithmetic that involve ordering, grouping and the numbers 1 and 0, to do simple computations with whole numbers.

### IV. DATA ANALYSIS, STATISTICS AND PROBABILITY

#### A. Data and Statistics

Standard: Represent and interpret data in real-world and mathematical problems.

The student will:

1. Collect data using observations or surveys and represent the data with tables and graphs with labeling.
2. Use mathematical language to describe a set of data.

#### B. Probability

Standard: Model simple probabilities by displaying the outcomes for real-world and mathematical problems.

The student will:

1. Express outcomes of random experiments verbally and numerically such as 3 out of 4 or  $\frac{3}{4}$ .
2. Use physical models and pictures to represent possible arrangements of two or three objects.

## V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT

### A. Spatial Sense

Standard: Understand spatial relationships and describe them using language such as congruent, similar, parallel and perpendicular.

The student will:

1. Identify congruent and similar figures.
2. Identify parallel and perpendicular lines.

### B. Geometry

Standard: Use attributes of two-and three-dimensional shapes to identify them and distinguish between them.

The student will:

1. Identify, describe, and classify two- and three-dimensional shapes by their attributes.
2. Identify right angles in geometric figures or in appropriate objects and determine whether other angles are greater or less than a right angle.

### C. Measurement

Standard: Measure and calculate length and area using appropriate tools and units to solve real-world and mathematical problems. Make change with money.

The student will:

1. Find the area and perimeter of a rectangle by measuring, using a grid, or using a formula, and label the answer with appropriate units.
2. Understand that rectangles with the same area can have different perimeters and that rectangles with the same perimeter can have different areas.
3. Make change using as few coins and bills as possible up to \$20.

## GRADE 5

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The student will:

1. Communicate, reason and represent situations mathematically.
2. Solve problems by distinguishing relevant from irrelevant information, sequencing and prioritizing information and breaking multi-step problems into simpler parts.
3. Evaluate the reasonableness of the solution by considering appropriate estimates and the context of the original problem.
4. Know when it is appropriate to estimate and when an exact answer with whole numbers, fractions or decimals is needed.
5. Express a written problem in suitable mathematical language, solve the problem and interpret the result in the original context.
6. Support mathematical results using pictures, numbers, and words to explain why the steps in a solution are valid and why a particular solution method is appropriate.
7. Organize, record and communicate math ideas coherently and clearly.

## II. NUMBER SENSE, COMPUTATION AND OPERATIONS

### A. Number Sense

Standard: Represent fractions, decimals and whole numbers in a variety of ways, to quantify information and to solve real-world and mathematical problems. Understand the concept of negative numbers.

The student will:

1. Read and write numbers up to three decimal places in numerals and words.
2. Represent and compare positive and negative integers symbolically and on the number line and use them to solve real-world and mathematical problems.
3. Recognize equivalent common fractions, decimals and percentages.
4. Use a variety of estimation strategies such as rounding, truncation, over- and underestimation and decide when an estimated solution is appropriate.

### B. Computation and Operation

Standard: Compute fluently and make reasonable estimates with fractions, decimals, and whole numbers, in real-world and mathematical problems. Understand the meanings of arithmetic operations and how they relate to one another.

The student will:

1. Use addition, subtraction, multiplication and division of multi-digit whole numbers to solve multi-step, real-world and mathematical problems.
2. Add and subtract numbers with up to two decimal places in real-world or mathematical problems.
3. Add and subtract, without a calculator, numbers containing up to five digits such as  $546.23 - 84.1$ .
4. Multiply, without a calculator, a two-digit whole number or decimal by a two-digit whole number or decimal, such as  $3.2 \times 3.4$ .
5. Multiply, without a calculator, a three-digit whole number or decimal by a one-digit whole number or decimal such as 3.51 divided by 3.



6. Model simple problems, arising from concrete situations, involving the addition and subtraction of common fractions and mixed numbers as well as fractions where the common denominator equals one of the denominators.
7. Interpret percents as a part of a hundred.

### III. PATTERNS, FUNCTIONS AND ALGEBRA

#### A. Patterns and Functions

Standard: Understand and describe patterns in numbers, shapes, tables and graphs.

The student will:

1. Identify patterns in numbers, shapes, tables, and graphs and explain how to extend those patterns.

#### B. Algebra (Algebraic Thinking)

Standard: Represent mathematical relationships using equations.

The student will:

1. Evaluate numeric expressions in real-world and mathematical problems.

### IV. DATA ANALYSIS, STATISTICS AND PROBABILITY

#### A. Data and Statistics

Standard: Represent data and use various measures associated with data to draw conclusions and identify trends.

The student will:

1. Determine whether or not a given graph matches a given data set.
2. Use fractions and percentages to compare data sets.
3. Collect data using measurements, surveys or experiments and represent the data with tables and graphs with labeling.
4. Find mean, mode, median, and range of a data set.

#### B. Probability

Standard: Model simple probabilities by displaying the outcomes for real-world and mathematical problems.

The student will:

1. Represent all possible outcomes for a simple probability problem with tables and grids, and draw conclusions from the results.

## V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT

### A. Spatial Sense

Standard: Understand the concepts of reflection and rotation symmetry as applied to two-dimensional shapes.

The student will:

1. Identify reflection and rotation symmetries in two-dimensional shapes and designs.

### B. Geometry

Standard: Sort, classify, compare and describe two- and three-dimensional objects.

The student will:

1. Sort three-dimensional objects according to number and shape of faces, number of edges and vertices.
2. Classify, compare and identify acute, right and obtuse angles.
3. Classify polygons as regular or irregular.
4. Know the sum of the angles in triangles and quadrilaterals.

### C. Measurement

Standard: Measure and calculate length, area and capacity using appropriate tools and units to solve real-world and mathematical problems.

The student will:

1. Find the area and perimeter of a triangle by measuring or using a grid, and label the answer with appropriate units.
2. Use a two-dimensional pattern of a cube or rectangular box to compute the surface area.
3. Select and apply the appropriate units and tools to measure perimeter, area and capacity.

## GRADE 6

These standards and benchmarks are to be used as a source document for identifying what all students should know and be able to do to demonstrate mathematical proficiency. To determine grade level placement of specific standards and benchmarks, judgment by experienced teachers was used to determine at what grade level 80% of children would master the specific material.

The current document identifies the grade at which mastery of each concept is expected but does not identify when those concepts are introduced and reinforced.. Schools must determine where in their curriculum these concepts would be introduced and reinforced so that they may be assessed at the indicated grade level.

Teachers must develop and enrich students' knowledge of mathematics beyond what is outlined in this document. It is critical for teachers to recognize the entire progression of standards and benchmarks before and after their grade level.

### I. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining four content strands.

*Note about assessment of this standard: The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.*

The student will:

1. Assess the reasonableness of a solution by comparing the solution to appropriate graphical or numerical estimates or by recognizing the feasibility of a solution in a given context.
2. Appropriately use examples and counterexamples to make and test conjectures, justify solutions and explain results.
3. Translate a problem described verbally or by tables, diagrams or graphs, into suitable mathematical language, solve the problem mathematically and interpret the result in the original context.
4. Support mathematical results by explaining why the steps in a solution are valid and why a particular solution method is appropriate.
5. Determine whether or not relevant information is missing from a problem.
6. Use accurately common logical words and phrases such as "and," "or," "if ... then ...," "unique," "only if."

## II. NUMBER SENSE, COMPUTATION AND OPERATIONS

### A. Number Sense

Standard: Use positive and negative rational numbers, represented in a variety of ways, to quantify information and to solve real-world and mathematical problems.

The student will:

1. Order and compare integers, fractions, decimals and mixed numbers with  $>$ ,  $<$ , and  $=$ . Locate and compare positive and negative rational numbers on a number line.
2. Use rounding and estimation with integers, decimals and fractions to solve real-world and mathematical problems.

### B. Computation and Operation

Standard: Compute fluently and make reasonable estimates with positive and negative rational numbers in real-world and mathematical problems. Understand the meanings of arithmetic operations and factorization, and how they relate to one another. Appropriately use calculators and other technologies to solve problems.

The student will:

1. Determine the prime factorization of positive integers.
2. Determine the least common multiple and the greatest common divisor of whole numbers.
3. Use addition, subtraction, multiplication and division of multi-digit whole and decimal numbers to solve multi-step real-world and mathematical problems.
4. Multiply and divide, without a calculator, numbers containing up to three digits by numbers containing up to two digits, such as  $347 / 83$  or  $4.91 \times 9.2$ .
5. Find quotients with remainders and be able to express the remainder in various ways depending on the context of the problem.
6. Use the relationship between moving the decimal point and the operations of multiplication or division by powers of ten to simplify calculations.
7. Add, subtract, multiply and divide common fractions and mixed numbers as well as fractions where the common denominator equals one of the denominators.

8. Find, represent and use percentages in real-world and mathematical problems, including percentages greater than 100% and less than 1%.
9. Apply the correct order of operations and grouping symbols when using calculators and other technologies.
10. Know, use and translate calculator notational conventions to mathematical notation.
11. Understand that use of a calculator requires appropriate mathematical reasoning and does not replace the need for mental computation.

### III. PATTERNS, FUNCTIONS AND ALGEBRA

#### A. Patterns and Functions

Standard: Demonstrate understanding of the rectangular coordinate system.

The student will:

1. Demonstrate understanding of the four quadrants in a rectangular coordinate system by writing and plotting ordered pairs.

#### B. Algebra (Algebraic Thinking)

Standard: Apply arithmetic operations in the correct order to simplify and evaluate numeric expressions in real-world and mathematical problems.

The student will:

1. Apply the correct order of operations including addition, subtraction, multiplication, division and grouping symbols to simplify and evaluate numeric expressions.

### IV. DATA ANALYSIS, STATISTICS AND PROBABILITY

#### A. Data and Statistics

Standard: Represent data and use various measures associated with data to draw conclusions and identify trends.

The student will:

1. Collect, organize and represent categorical and numerical data with tables and bar graphs.

2. Understand the differences and appropriate use of mean, median and mode.
3. Find the median and possible outliers.

### **B. Probability**

Standard: Calculate and express probabilities numerically, and apply probability concepts to solve real-world and mathematical problems.

The student will:

1. Generate and display data in graphs and tables to estimate experimental probabilities.
2. Represent all possible outcomes for a probability problem with tables, grids and tree diagrams to calculate probabilities and draw conclusions from the results.

## **V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT**

### **A. Spatial Sense**

Standard: Recognize the relationship between different representations of two- and three-dimensional shapes. Understand the effect of various transformations.

The student will:

1. Create models of three-dimensional geometric shapes from two-dimensional representations.
2. Predict the position and orientation of simple geometric shapes under transformations such as reflections, rotations and translations.
3. Identify symmetries in three-dimensional shapes.

### **B. Geometry**

Standard: Identify a variety of simple geometric figures by name, calculate various quantities associated with them and use appropriate tools to draw them.

The student will:

1. Use facts about angles including the relationship between complementary angles, supplementary angles and the angles within triangles to solve real-world and mathematical problems.
2. Classify triangles as equilateral, isosceles or scalene, and right, acute or obtuse.

3. Find the area and circumference of a circle given the radius or diameter using common approximations of pi where appropriate.
4. Measure, identify, and draw perpendicular and parallel lines, angles and rectangles by using appropriate tools such as straightedge, ruler, compass, protractor or software.

### **C. Measurement**

Standard: Make calculations of time, length, area and volume within standard measuring systems, using good judgment in choice of units.

The student will:

1. Solve problems requiring conversion of units within the U.S. customary system, and within the metric system.
2. Express measures of time and distance as fractions, mixed numbers and decimals to solve real-world and mathematical problems.
3. Find the area and perimeter of rectangles, squares, triangles and parallelograms by measuring, using a grid or using a formula.

## GRADE 7

These standards and benchmarks are to be used as a source document for identifying what all students should know and be able to do to demonstrate mathematical proficiency. To determine grade level placement of specific standards and benchmarks, judgment by experienced teachers was used to determine at what grade level 80% of children would master the specific material.

The current document identifies the grade at which mastery of each concept is expected but does not identify when those concepts are introduced and reinforced.. Schools must determine where in their curriculum these concepts would be introduced and reinforced so that they may be assessed at the indicated grade level.

Teachers must develop and enrich students' knowledge of mathematics beyond what is outlined in this document. It is critical for teachers to recognize the entire progression of standards and benchmarks before and after their grade level.

### I. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining four content strands.

*Note about assessment of this standard: The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.*

The student will:

1. Assess the reasonableness of a solution by comparing the solution to appropriate graphical or numerical estimates or by recognizing the feasibility of a solution in a given context.
2. Appropriately use examples and counterexamples to make and test conjectures, justify solutions and explain results.
3. Translate a problem described verbally or by tables, diagrams, or graphs, into suitable mathematical language, solve the problem mathematically and interpret the result in the original context.
4. Support mathematical results by explaining why the steps in a solution are valid and why a particular solution method is appropriate.
5. Determine whether or not relevant information is missing from a problem.
6. Use accurately common logical words and phrases such as “and,” “or,” “if ... then ...,” “unique,” “only if.”



## II. NUMBER SENSE, COMPUTATION AND OPERATIONS

### A. Number Sense

Standard: Use positive and negative rational numbers, represented in a variety of ways, to quantify information and to solve real-world and mathematical problems.

The student will:

1. Represent rational numbers as fractions, mixed numbers, decimals or percents and convert among various forms as appropriate.
2. Use scientific notation with positive powers of 10, with appropriate treatment of significant digits, to solve real-world and mathematical problems.
3. Locate and compare positive and negative rational numbers on a number line.

### B. Computation and Operation

Standard: Compute fluently and make reasonable estimates with rational numbers in real-world and mathematical problems. Understand the meanings of the basic operations, including the use of integer exponents and square roots, and how the operations relate to one another. Appropriately use calculators and other technologies to solve problems.

The student will:

1. Add, subtract, multiply and divide fractions and mixed numbers.
2. Use the inverse relationship between extracting square roots and squaring positive integers to solve real-world and mathematical problems.
3. Calculate the percentage of increase and decrease of a quantity in real-world and mathematical problems.
4. Convert among fractions, decimals and percents and use these representations for estimations and computations in real-world and mathematical problems.
5. Understand and compute positive integer powers of nonnegative integers and express examples as repeated multiplication such as  $3^4 = 3 \times 3 \times 3 \times 3 = 81$ .
6. Apply the correct order of operations and grouping symbols when using calculators and other technologies.
7. Know, use and translate calculator notational conventions to mathematical notation.

8. Understand that use of a calculator requires appropriate mathematical reasoning and does not replace the need for mental computation.

### III. PATTERNS, FUNCTIONS AND ALGEBRA

#### A. Patterns and Functions

Standard: Demonstrate an understanding of rate of change graphically and numerically.

The student will:

1. Demonstrate, numerically and graphically, an understanding that rate is a measure of change of one quantity per unit change of another quantity in real-world and mathematical problems.
2. Plot points on the graph of a linear function and identify the slope or rate of change.

#### B. Algebra (Algebraic Thinking)

Standard: Apply arithmetic operations in the correct order to generate equivalent algebraic expressions and to solve simple formulas in real-world and mathematical problems.

The student will:

1. Apply the correct order of operations including addition, subtraction, multiplication, division and grouping symbols to generate equivalent algebraic expressions.
2. Use the facts that the sum of a number and its opposite is zero and the product of a number and its reciprocal is one to generate equivalent algebraic expressions.
3. Solve simple formulas with up to three variables, when the values of two of the variables are given.

### IV. DATA ANALYSIS, STATISTICS AND PROBABILITY

#### A. Data and Statistics

Standard: Represent data and use various measures associated with data to draw conclusions and identify trends.

The student will:

1. Construct and analyze simple scatter plots.
2. Understand the meaning of, and be able to compute minimum, maximum, range, median, mean and mode of a data set.

**B. Probability**

Standard: Calculate and express probabilities numerically and apply probability concepts to solve real-world and mathematical problems.

The student will:

1. Express probabilities as percentages, fractions, proportions and decimals.
2. Use a variety of experiments to explore the relationship between experimental and theoretical probabilities.

**V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT****A. Spatial Sense**

Standard: Recognize the relationship between different representations of two- and three-dimensional shapes. Understand the effect of various transformations.

The student will:

1. Recognize a view of a three-dimensional shape, given a view from a different orientation.
2. Use visual representations of transformations such as reflections, rotations, translations and change of scale in one and two dimensions to solve real-world and mathematical problems.

**B. Geometry**

Standard: Use basic geometric principles and proportional reasoning to solve real-world and mathematical problems.

The student will:

1. Calculate the radius, diameter, circumference and area of a circle given any one of these.
2. Calculate the area and perimeter of a sector of a circle given its angle and radius.
3. Use ratios and proportions to interpret map scales and scale drawings.
4. Classify quadrilaterals as squares, rectangles, rhombi, parallelograms, kites, trapezoids or none of these.

**C. Measurement**

Standard: Make calculations of time, length, area and volume within standard measuring systems using good judgment in choice of units.

The student will:

1. Choose appropriate units to calculate, measure, and record length, weight, area and volume in both U.S. customary and metric systems.

## GRADE 8

These standards and benchmarks are to be used as a source document for identifying what all students should know and be able to do to demonstrate mathematical proficiency. To determine grade level placement of specific standards and benchmarks, judgment by experienced teachers was used to determine at what grade level 80% of children would master the specific material.

The current document identifies the grade at which mastery of each concept is expected but does not identify when those concepts are introduced and reinforced.. Schools must determine where in their curriculum these concepts would be introduced and reinforced so that they may be assessed at the indicated grade level.

Teachers must develop and enrich students' knowledge of mathematics beyond what is outlined in this document. It is critical for teachers to recognize the entire progression of standards and benchmarks before and after their grade level.

### I. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining four content strands.

*Note about assessment of this standard: The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.*

The student will:

1. Assess the reasonableness of a solution by comparing the solution to appropriate graphical or numerical estimates or by recognizing the feasibility of a solution in a given context.
2. Appropriately use examples and counterexamples to make and test conjectures, justify solutions and explain results.
3. Translate a problem described verbally or by tables, diagrams or graphs, into suitable mathematical language, solve the problem mathematically and interpret the result in the original context.
4. Support mathematical results by explaining why the steps in a solution are valid and why a particular solution method is appropriate.
5. Determine whether or not relevant information is missing from a problem.
6. Use accurately common logical words and phrases such as “and,” “or,” “if ... then ...,” “unique,” “only if.”

## II. NUMBER SENSE, COMPUTATION AND OPERATIONS

### A. Number Sense

Standard: Use rational and irrational numbers, represented in a variety of ways, to quantify information and to solve real-world and mathematical problems.

The student will:

1. Represent and compare rational and irrational numbers symbolically and on a number line.
2. Use rational and irrational numbers to solve real-world and mathematical problems.
3. Use scientific notation with positive and negative powers of 10, with appropriate treatment of significant digits, to solve real-world and mathematical problems.
4. Classify numbers as rational or irrational.

### B. Computation and Operation

Standard: Compute fluently and make reasonable estimates with rational and irrational numbers in real-world and mathematical problems. Understand the meanings of the basic operations, including the use of integer exponents and  $n^{\text{th}}$  roots, and how the operations relate to one another. Appropriately use calculators and other technologies to solve problems.

The student will:

1. Use calculator approximations of irrational and rational numbers in multi-step real-world and mathematical problems.
2. Find integer approximations of square roots of positive integers without a calculator.
3. Multiply and divide expressions involving exponents with a common base.
4. Use the inverse relationship between  $n^{\text{th}}$  roots and  $n^{\text{th}}$  powers of rational numbers to solve real-world and mathematical problems.
5. Apply the correct order of operations and grouping symbols when using calculators and other technologies.
6. Know, use and translate calculator notational conventions to mathematical notation.
7. Understand that use of a calculator requires appropriate mathematical reasoning and does not replace the need for mental computation.

### III. PATTERNS, FUNCTIONS AND ALGEBRA

#### A. Patterns and Functions

Standard: Understand and describe progressions. Use graphs and tables to solve real-world and mathematical problems.

The student will:

1. Recognize when a list of numbers forms an arithmetic or geometric progression and be able to determine subsequent terms in the progression.
2. Represent quantitative relationships graphically and use the graphs to solve real-world and mathematical problems.
3. Generate a table of values from a formula and graph the resulting ordered pairs on a grid.

#### B. Algebra (Algebraic Thinking)

Standard: Use algebraic operations to generate equivalent expressions, and use proportional reasoning to solve real-world and mathematical problems. Demonstrate the ability to manipulate an equation by applying arithmetic operations to both sides to maintain equivalence.

The student will:

1. Multiply and divide expressions of the form  $ax^n$ .
2. Use simple formulas with more than one variable to solve real-world and mathematical problems.
3. Use proportions and percents with one unknown quantity to solve real-world and mathematical problems.
4. Apply the correct order of operations including addition, subtraction, multiplication, division, grouping symbols and powers, to simplify and evaluate algebraic expressions.

### IV. DATA ANALYSIS, STATISTICS, AND PROBABILITY

#### A. Data and Statistics

Standard: Represent data and use various measures associated with data to draw conclusions and identify trends.

The student will:

1. Construct and analyze histograms, circle graphs, stem-and-leaf plots and box-and-whisker plots.
2. Compute the quartiles of a data set.

### **B. Probability**

Standard: Calculate and express probabilities numerically and apply probability concepts to solve real-world and mathematical problems.

The student will:

1. Understand that if  $p$  is the probability of an event occurring, then  $1 - p$  is the probability of the event not occurring.
2. Convert between odds and probabilities.
3. Use a variety of experiments to explore the relationship between experimental and theoretical probabilities and the effect of sample size on this relationship.

## **V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT**

### **A. Spatial Sense**

Standard: Recognize the relationship between different representations of two- and three-dimensional shapes. Understand the effect of various transformations.

The student will:

1. Use models and visualization to understand and create various two-dimensional diagrams of three-dimensional shapes.
2. Predict the position and orientation of simple three-dimensional geometric shapes under transformations such as reflections, rotations and translations.

### **B. Geometry**

Standard: Use basic geometric principles and proportional reasoning to solve real-world and mathematical problems.

The student will:



1. Apply the relationship between changes in one or more linear distances in a planar figure and the change in area.
2. Use the concept of similarity in simple two-dimensional figures to solve real-world and mathematical problems involving proportionality.
3. Know how to find the volumes of cubes, prisms, spheres and cylinders.
4. Know how to find the surface areas of cubes, prisms and cylinders.
5. Calculate perimeter and area of two-dimensional figures obtained by putting together triangles, parallelograms, and sectors of circles to solve real-world and mathematical problems.

### **C. Measurement**

Standard: Make calculations of time, length, area and volume within and between standard measuring systems using good judgment in choice of units.

The student will:

1. Find approximate equivalent measures of length, temperature and weight for common units in U.S. customary and metric measuring systems.
2. Use arithmetic to solve simple real-world and mathematical problems involving mixed units such as minutes and hours in elapsed time, degrees and minutes in latitude and longitude and feet and inches in distance.

## GRADES 9, 10, 11

These standards and benchmarks are to be used as a source document for identifying what all students should know and be able to do to demonstrate mathematical proficiency. To determine grade level placement of specific standards and benchmarks, judgment by experienced teachers was used to determine at what grade level 80% of children would master the specific material.

Teachers must develop and enrich students' knowledge of mathematics beyond what is outlined in this document. It is critical for teachers to recognize the entire progression of standards and benchmarks before and after their grade level.

The academic standards for grades 9, 10 and 11 have been grouped together to allow for flexibility within the individual districts.

### D. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining three content strands.

*Note about assessment of this standard: The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.*

The student will:

1. Assess the reasonableness of a solution by comparing the solution to appropriate graphical or numerical estimates or by recognizing the feasibility of solutions in a given context and rejecting extraneous solutions.
2. Appropriately use examples and counterexamples to make and test conjectures, justify solutions, and explain results.
3. Translate a problem described verbally or by tables, diagrams or graphs, into suitable mathematical language, solve the problem mathematically and interpret the result in the original context.
4. Support mathematical results by explaining why the steps in a solution are valid and why a particular solution method is appropriate.
5. Determine whether or not relevant information is missing from a problem and if so, decide how to best express the results that can be obtained without that information.
6. Know and use the relationship that exists among a logical implication of the form "if A, then B," its converse "if B, then A," its inverse "if not A, then not B," and its contrapositive "if not B, then not A."

## II. NUMBER SENSE, COMPUTATION, AND OPERATIONS

### A. Number Sense

Standard: Use real numbers, represented in a variety of ways, to quantify information and to solve real-world and mathematical problems.

### B. Computation and Operation

Standard: Appropriately use calculators and other technologies to solve algebraic, geometric, probabilistic and statistical problems.

The student will:

1. Apply the correct order of operations and grouping symbols when using calculators and other technologies.
2. Know, use and translate calculator notational conventions to mathematical notation.
3. Recognize the impact of units such as degrees and radians on calculations.
4. Recognize that applying an inverse function with a calculator may lead to extraneous or incomplete solutions.
5. Understand the limitations of calculators such as missing or additional features on graphs due to viewing parameters or misleading representations of zero or very large numbers.
6. Understand that use of a calculator requires appropriate mathematical reasoning and does not replace the need for mental computation.

## III. PATTERNS, FUNCTIONS, AND ALGEBRA

### A. Patterns and Functions

Standard: Represent and analyze real-world and mathematical problems using numeric, graphic and symbolic methods for a variety of functions.

The student will:

1. Know the numeric, graphic and symbolic properties of linear, step, absolute value and quadratic functions. Graphic properties may include rates of change, intercepts, maxima and minima.

2. Model exponential growth and decay, numerically, graphically and symbolically, using exponential functions with integer inputs.
3. Analyze the effects of coefficient changes on linear and quadratic functions and their graphs.
4. Apply basic concepts of linear, quadratic and exponential expressions or equations in real-world problems such as loans, investments and the path of a projectile.
5. Distinguish functions from other relations using graphic and symbolic methods.

### **B. Algebra (Algebraic Thinking)**

Standard: Solve simple equations and inequalities numerically, graphically, and symbolically. Use recursion to model and solve real-world and mathematical problems.

The student will:

1. Translate among equivalent forms of expressions, such as, simplify algebraic expressions involving nested pairs of parentheses and brackets, simplify rational expressions, factor a common term from an expression and apply associative, commutative and distributive laws.
2. Understand the relationship between absolute value and distance on the number line and graph simple expressions involving absolute value such as,  $|x - 3| = 6$  or  $|x + 2| < 5$ .
3. Find equations of a line given two points on the line, a point and the slope of the line or the slope and the y-intercept of the line.
4. Translate among equivalent forms of linear equations and inequalities.
5. Use a variety of models such as equations, inequalities, algebraic formulas, written statements, tables and graphs or spreadsheets to represent functions and patterns in real-world and mathematical problems.
6. Apply the laws of exponents to perform operations on expressions with integer exponents.
7. Solve linear equations and inequalities in one variable with numeric, graphic and symbolic methods.
8. Find real solutions to quadratic equations in one variable with numeric, graphic and symbolic methods.

9. Use appropriate terminology and mathematical notation to define and represent recursion.
10. Create and use recursive formulas to model and solve real-world and mathematical problems.
11. Solve systems of two linear equations and inequalities with two variables using numeric, graphic and symbolic methods.
12. Understand how slopes can be used to determine whether lines are parallel or perpendicular. Given a line and a point not on the line, find the equations for the lines passing through that point and parallel or perpendicular to the given line.

#### **IV. DATA ANALYSIS, STATISTICS, AND PROBABILITY**

##### **A. Data and Statistics**

Standard: Represent data and use various measures associated with data to draw conclusions and identify trends. Understand the effects of display distortion and measurement error on the interpretation of data.

The student will:

1. Construct and analyze circle graphs, bar graphs, histograms, box-and-whisker plots, scatter plots and tables, and demonstrate the strengths and weaknesses of each format by choosing appropriately among them for a given situation.
2. Use measures of central tendency and variability, such as, mean, median, maximum, minimum, range, standard deviation, quartile and percentile, to describe, compare and draw conclusions about sets of data.
3. Determine an approximate best-fit line from a given scatter plot and use the line to draw conclusions.
4. Know the influence of outliers on various measures and representations of data about real-world and mathematical problems.
5. Understand the relationship between correlation and causation.
6. Interpret data credibility in the context of measurement error and display distortion.
7. Compare outcomes of voting methods such as majority, plurality, ranked by preference, run-off and pair-wise comparison.

**B. Probability**

Standard: Use appropriate counting procedures, calculate probabilities in various ways and apply theoretical probability concepts to solve real-world and mathematical problems.

The student will:

1. Select and apply appropriate counting procedures to solve real-world and mathematical problems, including probability problems.
2. Use area, trees, unions and intersections to calculate probabilities and relate the results to mutual exclusiveness, independence and conditional probabilities, in real-world and mathematical problems.
3. Use probability models, including area and binomial models, in real-world and mathematical problems.
4. For simple probability models, determine the expected values of random variables.
5. Know the effect of sample size on experimental and simulation probabilities.
6. Use a variety of experimental, simulation and theoretical methods to calculate probabilities.

**V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT****A. Spatial Sense**

Standard: Use models to represent and understand two- and three-dimensional shapes and how various motions affect them. Recognize the relationship between different representations of the same shape.

The student will:

1. Use models and visualization to understand and represent three-dimensional objects and their cross sections from different perspectives.

**B. Geometry**

Standard: Apply basic theorems of plane geometry, right triangle trigonometry, coordinate geometry and a variety of visualization tools to solve real-world and mathematical problems.

The student will:

1. Know and use theorems about triangles and parallel lines in elementary geometry to justify facts about various geometrical figures and solve real-world and mathematical problems. These theorems include criteria for two triangles to be congruent or similar and facts about parallel lines cut by a transversal.
2. Know and use theorems about circles to justify geometrical facts and solve real-world and mathematical problems. These theorems include the relationships involving tangent lines and radii, the relationship between inscribed and central angles and the relationship between the measure of a central angle and arc length.
3. Know and use properties of two- and three-dimensional figures to solve real-world and mathematical problems such as: finding area, perimeter, volume and surface area; applying direct or indirect methods of measurement; the Pythagorean theorem and its converse; and properties of  $45^\circ$ - $45^\circ$ - $90^\circ$  and  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles.
4. Apply the basic concepts of right triangle trigonometry including sine, cosine and tangent to solve real-world and mathematical problems.
5. Use coordinate geometry to represent and examine geometric concepts such as the distance between two points, the midpoint of a line segment, the slope of a line and the slopes of parallel and perpendicular lines.
6. Use numeric, graphic and symbolic representations of transformations such as reflections, translations and change of scale in one, two and three dimensions to solve real-world and mathematical problems.
7. Perform basic constructions with a straightedge and compass.
8. Draw accurate representations of planar figures using a variety of tools.

### **C. Measurement**

Standard: Use the interconnectedness of geometry, algebra and measurement to explore real-world and mathematical problems.

## GRADES 11-12

In any classroom there may be students choosing more electives in mathematics or taking those electives at an earlier grade than their classmates. These standards and benchmarks are to be used as a source document for identifying possible avenues to address the needs of those students.

It is critical for teachers to recognize the entire progression of standards and benchmarks before and after their grade level. Teachers must develop and enrich students' knowledge of mathematics as appropriate for each child.

### STATISTICS

Standard: Use tables of the normal distribution and properties of that distribution to make judgments about populations based on random samples from these populations.

The student will:

1. Use the concept of normal distribution and its properties to answer questions about sets of data.
2. Describe and use sampling distributions and the central limit theorem. Calculate confidence intervals when appropriate.
3. Understand the importance of appropriate sampling methods. For instance, the time of day of a survey could lead to inaccuracies in the outcome.

### ALGEBRA

Standard: Demonstrate facility with a wide range of algebraic operations and use the relationship between coordinate geometry and algebraic equations to solve real-world and mathematical problems.

The student will:

1. Solve systems of two, three or more simultaneous linear equations or inequalities, in particular, deciding whether a given system of equations has one solution, no solution or infinitely many solutions and, in this latter case, describing them parametrically.
2. Solve problems with quadratic functions and equations, where some of the coefficients may be expressed in terms of parameters.
3. Perform the four arithmetic operations with polynomials, except that division is restricted to division by monomials and linear binomials.
4. Simplify a wide variety of algebraic expressions, including those in which numerator or denominator needs to be rationalized.



5. Apply the laws of exponents to perform operations on expressions with fractional exponents.
6. Know the numeric, graphic and symbolic properties of power, logarithmic and exponential functions.
7. Solve a wide variety of mathematical and real-world problems involving power, exponential and logarithmic functions and equations, discard extraneous solutions and present results graphically.
8. Know the numeric, graphic and symbolic properties of rational functions.
9. Solve a wide variety of mathematical and real-world problems involving rational functions, discard extraneous solutions and present results graphically.
10. Factor polynomials representing the difference of squares, perfect square trinomials and quadratics with rational factors.
11. Make sketches including axes, centers, asymptotes, vertices of parabola, ellipses (including circles) and hyperbolas with axes parallel to the coordinate axes, given their equations, and completing the square if necessary.
12. Find equations of parabolas, ellipses and hyperbolas when presented with their graphs having axes parallel to the coordinate axes.
13. Add, subtract, multiply and divide complex numbers, interpret sums geometrically, and find complex solutions of quadratic equations.
14. Know and use the Factor and Remainder Theorems.
15. Find the inverse of a function and the composition of functions by numeric and symbolic methods. Know the relationship between the graphs of a function and its inverse.
16. Know and use formal notation for sequences and series to solve related problems.

## TRIGONOMETRY & GEOMETRY

Standard: Understand the properties of the standard trigonometric functions and apply them to real-world and mathematical problems, especially geometrical problems. Develop increased mastery of geometric proof methodology.

The student will:

1. Know the six trigonometric functions defined for an angle in a right triangle.

2. Given the coordinates of a point on the terminal side of an angle in standard position in the  $xy$ -plane, find the values of the trigonometric functions.
3. Convert between degrees and radian measures.
4. Solve applied problems about triangles using the law of sines including the ambiguous case.
5. Solve applied problems about triangles using the law of cosines.
6. Graph the functions of the form  $A\sin(Bt + C)$ ,  $A\cos(Bt + C)$ , and  $A\tan(Bt + C)$  and know the meaning of the terms frequency, amplitude, phase shift and period.
7. Simplify trigonometric expressions using identities and verify simple trigonometric identities including  $\sin^2x + \cos^2x = 1$ , sum, difference, double angle and half-angle formulas for sine and cosine.
8. Find all the solutions of a trigonometric equation on various intervals.
9. Know and be able to use the definitions of the inverse trigonometric functions and related methods to solve problems such as find  $\cos(x)$  and  $\tan(x)$  given the value of  $\sin x$  and the quadrant containing the terminal side.